SAWTIMBER VOLUME AND VOLUME-BASAL AREA RATIO EQUATIONS FOR RED PINE IN MICHIGAN: 9.6-IN. MINIMUM TOP DIAMETER

BY

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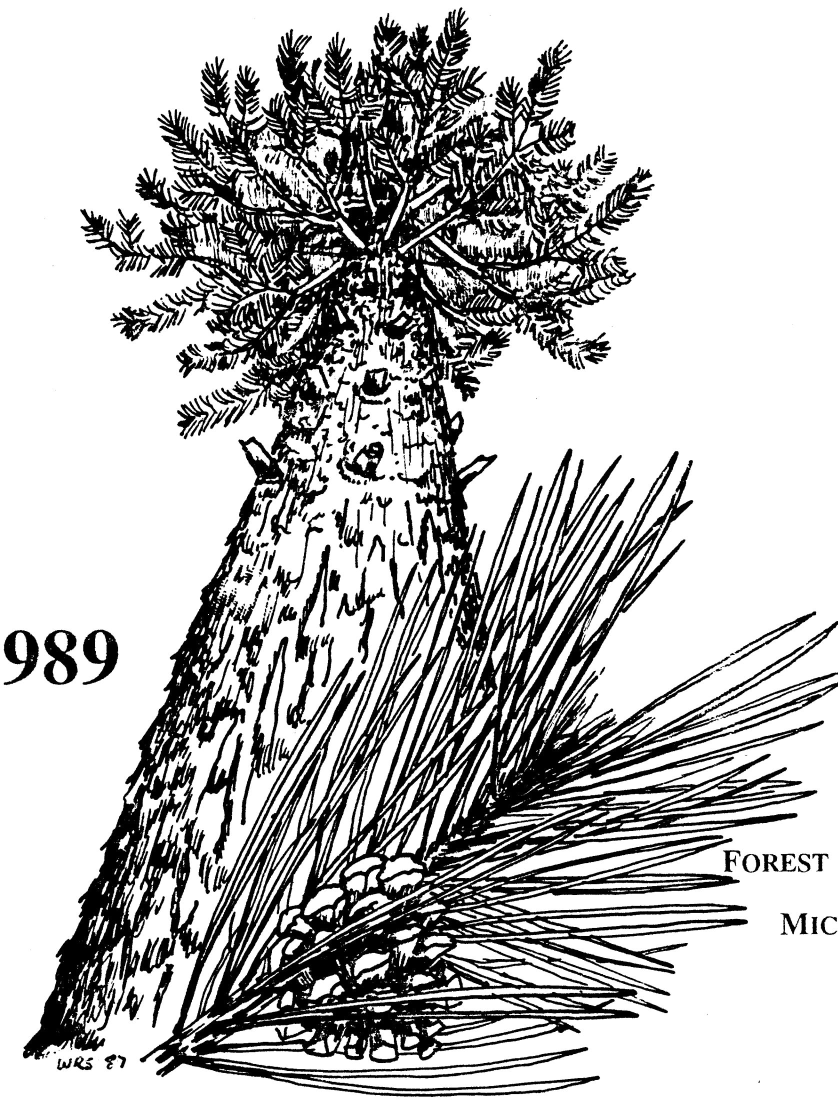

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Management Summary

New cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot individual tree sawtimber volume and volume-basal area ratio (VBAR) equations for a 9.6-in. minimum inside bark top diameter were developed for red pine in Michigan. Data used to develop these equations were collected from 27 red pine stands in Michigan (16 and 11 stands in the Upper and Lower Peninsulas, respectively).

Examination of coefficients of determination (R^2), standard errors of the estimate ($s_{y.x}$), and sample error terms indicated that (1) nonlinear equations were most accurate for all types of individual tree volumes, and (2) linear equations were most accurate for all types of VBARs. VBAR multiple linear regression equations using diameter at breast height (DBH) and merchantable height (MH) independent variables yielded somewhat more accuracy than VBAR equations using MH independent variables. However, the difference between the 2 sets of equations are relatively small, indicating that the use of the simpler height VBAR equations is justified for most cruising situations.

The new individual tree volume equations yielded volume estimates larger than the values in Tables 2-5 of Fowler and Hussain (1989a) for a 7.6-in. minimum top diameter with the difference increasing and decreasing with increasing MH and DBH, respectively. The new VBAR equations yielded VBAR estimates, in general, larger than the values in Table 2 of Fowler and Hussain (1989b) for a 7.6-in. minimum top diameter with the difference, in general, increasing and decreasing with increasing MH for cu.ft. and bd.ft. VBARs, respectively.

The new individual tree sawtimber volume equations are:

1. Cubic-foot volume

$$\hat{V}_C=0.1367D^{1.526}H^{0.8878}$$

2. Doyle board-foot volume

$$\hat{V}_D=0.1404D^{2.020}H^{0.9714}$$

3. International 1/4-inch board-foot volume

$$\hat{V}_I=0.6381D^{1.582}H^{0.9652}$$

4. Scribner board-foot volume

$$\hat{V}_S=0.5594D^{1.619}H^{0.9669}$$

where D is DBH in inches and H is MH in 100-in. sticks to a 9.6-in. minimum top diameter. Multiple linear regression equations were developed to predict (1) one type of volume from another type, and (2) Doyle, International 1/4-inch, and Scribner board-foot cubic-foot ratios as a function of D, H, and D and H.

We recommend the use of the following VBAR equations in most cruising situations for red pine:

1. Cubic-foot VBAR

$$\hat{VBAR}_C=9.706+3.822 \cdot H-5.641 \cdot \frac{1}{H}$$

2. Doyle board-foot VBAR

$$\hat{VBAR}_D=5.079+25.131 \cdot H-3.046 \cdot \frac{1}{H}$$

3. International 1/4-inch board-foot VBAR

$$\hat{VBAR}_I=41.653+26.908 \cdot H-25.239 \cdot \frac{1}{H}$$

4. Scribner board-foot VBAR

$$\hat{VBAR}_S=37.344+26.839 \cdot H-22.756 \cdot \frac{1}{H}$$

where H is the MH in 100-in. sticks to a 9.6-in. minimum top diameter. Multiple linear regression equations were also developed to predict one type of VBAR from another for the 4 types of VBARs examined in this study.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

Michigan Department of Natural Resources
Forest Management Division

SUBJECT -- INDIVIDUAL TREE VOLUME AND VOLUME-BASAL AREA RATIO EQUATIONS

DATE -- 16 Dec. 88

TITLE -- Sawtimber Volume and Volume-Basal Area Ratio Equations for Red Pine in Michigan: 9.6-in. Minimum Top Diameter

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Background

Composite individual tree sawtimber Scribner and International 1/4-inch board-foot volume tables have been developed for the Lake States by Gevorkiantz and Olsen (1955). The Michigan Department of Natural Resources (MDNR) developed sawtimber volume-basal area ratios (VBARs) in bd.ft./sq.ft. to be used in estimating International 1/4-inch board-foot volume using prism cruising. DNR Tally Sheet R 4145 was developed from these VBARs. Fowler and Hussain (1989a,b) developed sawtimber volume and volume-basal area ratio equations for red pine in Michigan. All of the above sawtimber equations assume an approximate 7.6-in. minimum top diameter.

Purpose

The MDNR needs sawtimber volume and VBAR equations for the Upper Peninsula for a 9.6-in. minimum top diameter. The purpose of this paper is to present new sawtimber cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot volume and VBAR equations and tables for red pine in Michigan for a 9.6-in. minimum top diameter.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 3507 trees from 27 stands as follows:

1) 2341 trees from 16 stands in the Upper Peninsula (i.e., 5, 4, and 7 stands in the eastern, central, and western U.P., respectively), and
2) 1166 trees from 11 stands in the Lower Peninsula (i.e., 4, 4, and 3 stands from the northeast, northwest, and southern L.P., respectively).

Stands were selected from the above 6 regions to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May-August, 1983-1985.

For the 27 stands, site index varied from 45 to 75, age varied from 26 to 105 years, basal area/acre varied from 90 to 225 sq.ft., average DBH varied from 6.7 to 17.7 in., average total height varied from 33.2 to 86.0 ft., and average merchantable height (MH) to an approximate 3.6-in. minimum top diameter varied from 2.0 to 8.4 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., MH to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and out-side (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5 ft.), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987a). Fowler and Hussain (1987b,c) developed pulpwood, sawtimber, and residual pulpwood cubic-foot volume and VBAR equations from the total data set described above. Fowler and Hussain (1989a,b) also developed sawtimber volume and VBAR cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot equations for a 7.6-in. minimum top diameter from the total data set described above.

Sawtimber trees were defined as trees that had at least one 100-in. stick with a minimum inside bark top diameter no smaller than 9.6 inches. Sawtimber MH is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 9.6 inches. There was a total of 1349 sawtimber trees.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into 2 pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the 2 volumes. For each 100-in. stick, cubic-foot and board-foot volumes were calculated using the following formulas:

Cubic-foot: $V=\frac{(B+b)L}{2}$ (Avery and Burkhart 1983)

Doyle: $V=0.5D^2-4.0D+8.0$ (Husch et al. 1982)

International 1/4-inch: $V=0.905(0.44D^2-1.20D-0.30)$ (Husch et al. 1982)

Scribner: $V=0.395D^2-0.99D-2.15$ (Bruce and Schumacher 1950) where

V=volume in cubic feet or board feet,

L=length of stick (100-in.) in ft.,

B=cross-sectional area inside bark of large end of the stick in sq.ft.,

b=cross-sectional area inside bark of small end of the stick in sq.ft., and

D=diameter of small end of the stick inside bark in inches.

See Avery and Burkhart (1983) and Husch et al. (1982) for detailed discussions of cubic-foot volumes and board-foot log rules.

Cubic-foot and the 3 board-foot volumes for each tree were determined by summing up the volumes of all sawtimber sticks to a 9.6-in. top diameter limit. Sawtimber VBARs were obtained for each tree by dividing cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot volumes of the tree by the basal area in sq.ft. of the tree at 4.5 ft. from the ground.

Sawtimber volume was regressed on various independent variables determined from tree DBH and MH using multiple linear and nonlinear regression. VBAR was regressed on various forms of sawtimber DBH and MH using multiple linear regression.

Results

The data set used to develop the regression equations consisted of 1349 trees. Fowler and Hussain (1987b) found no significant differences between the pulpwood cubic-foot volume equations of the 6 regions, so the data from all stands were pooled before developing volume and VBAR equations. All equations were based on the 1349 trees with an average DBH=13.9 in.

(range: 9.8 to 23.9), average MH=3.2 sticks (range: 1 to 7), average cubic-foot volume=24.02 (range: 4.24 to 94.91), average Doyle board-foot volume=109.2 (range: 15.4 to 542.2), average International 1/4-inch board-foot volume=145.94 (range: 25.67 to 621.84), average Scribner board-foot volume=142.13 (range: 24.42 to 613.51), average cubic-foot VBAR=19.28 cu.ft./sq.ft. (range: 7.16 to 38.23), average Doyle board-foot VBAR=83.47 bd.ft./sq.ft. (range: 22.16 to 200.96), average International 1/4-inch board-foot VBAR=115.65 bd.ft./sq.ft. (range: 35.66 to 243.05), and average Scribner board-foot VBAR=112.24 bd.ft./sq.ft. (range: 34.08 to 237.60).

Sawtimber Volume Equations

Cubic-foot and board-foot volume prediction equations

A comparison of various multiple linear regression and non-linear regression equations based on goodness-of-fit and simplicity indicated that the following nonlinear prediction equation compared favorably to all other equations examined for cubic-foot and the 3 board-foot volumes:

$$\hat{V} = \hat{\beta}_0 D^{\hat{\beta}_1} H^{\hat{\beta}_2}$$

where $\hat{V}$ is predicted volume, D is DBH in inches, and H is MH in 100-in sticks to a 9.6-in. top diameter limit. $\hat{\beta}_0$ is the sample intercept or regression constant, and $\hat{\beta}_1$ and $\hat{\beta}_2$ are the sample regression coefficients related to the independent variables.

Table 1 shows the sawtimber volume equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot

Table 1. Estimated parameters ($\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$), and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and 3 board-foot volumes.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2	$s_{y \cdot x}$
(1) Cubic-foot[a]	0.1367	1.526	0.8878	0.993	1.52
(2) Doyle[b]	0.1404	2.020	0.9714	0.983	12.72
(3) International 1/4-inch[c]	0.6381	1.582	0.9652	0.987	13.09
(4) Scribner[d]	0.5594	1.619	0.9669	0.987	13.08

[a] $\hat{V}=0.1367D^{1.526}H^{0.8878}$

[b] $\hat{V}=0.1404D^{2.020}H^{0.9714}$

[c] $\hat{V}=0.6381D^{1.582}H^{0.9652}$

[d] $\hat{V}=0.5594D^{1.619}H^{0.9669}$

volumes along with coefficients of determination (R^2) and standard errors of the estimate ($s_{y \cdot x}$).

A cubic-foot volume table is shown in Table 2, and Doyle, International 1/4-inch, and Scribner board-foot volume tables are shown in Tables 3, 4, and 5, respectively.

The values shown in Table 2 (cu.ft. volumes) are larger than the values in Table 2 of Fowler and Hussain (1989a), varying from 6-11% higher for DBH=11 in. to about 2-4% higher for DBH=25 in.

The values shown in Table 3 (Doyle bd.ft. volumes) are larger than the values in Table 3 of Fowler and Hussain (1989a), varying from 25-29% higher for DBH=11 in. to 1-2% higher for DBH=25 in.

The values shown in Table 4 (International 1/4-inch bd.ft. volumes) are larger than those values in Table 4 of Fowler and Hussain (1989a), varying from 12-19% higher for DBH=11 in. to 2-4% higher for DBH=25 in.

The values shown in Table 5 (Scribner bd.ft. volumes) are larger than those values in Table 5 of Fowler and Hussain (1989a), varying from 17-20% higher for DBH=11 in. to 2-4% higher for DBH=25 in.

For all 4 types of volumes, values for a 9.6-in. minimum top diameter were larger than values for a 7.6-in. minimum top diameter, with the difference increasing and decreasing with increasing MH and DBH, respectively.

Table 2. Volume table showing cu.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 1).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	5.3	9.8	14.1	18.2	22.2	26.0			
12	6.1	11.2	16.1	20.8	25.3	29.7	34.1		
13	6.8	12.7	18.2	23.5	28.6	33.6	38.5	43.4	
14	7.7	14.2	20.3	26.3	32.0	37.6	43.2	48.6	
15	8.5	15.8	22.6	29.2	35.6	41.8	47.9	54.0	59.9
16		17.4	24.9	32.2	39.2	46.1	52.9	59.6	66.1
17		19.1	27.4	35.3	43.1	50.6	58.0	65.3	72.5
18		20.8	29.8	38.5	47.0	55.2	63.3	71.3	79.2
19		22.6	32.4	41.8	51.0	60.0	68.8	77.4	86.0
20		24.5	35.1	45.3	55.2	64.9	74.4	83.7	93.0
21			37.8	48.8	59.4	69.9	80.1	90.2	100.2
22			40.5	52.3	63.8	75.0	86.0	96.8	107.5
23			43.4	56.0	68.3	80.3	92.1	103.6	115.1
24			46.3	59.8	72.9	85.7	98.2	110.6	122.8
25			49.3	63.6	77.5	91.2	104.5	117.7	130.7

Table 3. Volume table showing Doyle bd.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 2).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	18	35	52	69	85	102			
12	21	42	62	82	101	121	141		
13	25	49	73	96	119	142	165	188	
14	29	57	84	116	139	165	192	219	
15	33	65	97	128	159	190	221	251	282
16		74	110	146	181	217	252	286	321
17		84	125	165	205	245	284	324	363
18		95	140	185	230	275	319	363	407
19		105	156	207	257	306	356	405	454
20		117	173	229	285	340	395	449	504
21			191	253	314	375	436	496	556
22			210	278	345	412	479	545	611
23			230	304	378	451	524	596	668
24			251	331	412	491	571	650	728
25			272	360	447	533	620	705	791

Table 4. Volume table showing International 1/4-inch bd.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 3).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	28	55	82	108	134	160			
12	33	63	94	124	154	183	213		
13	37	72	107	141	174	208	241	275	
14	42	81	120	158	196	234	271	309	
15	46	90	134	176	219	261	303	344	386
16		100	148	195	242	289	335	381	427
17		110	163	215	267	318	369	420	470
18		121	178	235	292	348	404	460	515
19		131	194	256	318	379	440	501	561
20		142	211	278	345	411	477	543	608
21			228	300	373	444	516	587	657
22			245	323	401	478	555	631	707
23			263	347	430	513	595	677	759
24			281	371	460	549	637	725	812
25			300	396	491	585	679	773	866

Table 5. Volume table showing Scribner bd.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 4).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	27	53	79	104	129	154			
12	31	61	90	119	148	177	205		
13	36	70	103	136	169	201	234	266	
14	40	78	116	153	190	227	263	300	
15	45	88	130	171	213	254	294	335	375
16		97	144	190	236	282	327	372	417
17		107	159	210	260	311	361	410	460
18		118	174	230	286	341	395	450	504
19		129	190	251	312	372	432	491	550
20		140	207	273	339	404	469	534	598
21			224	295	367	437	508	578	647
22			241	319	395	471	547	623	698
23			259	342	425	507	588	669	750
24			278	367	455	543	630	717	803
25			297	392	486	580	673	766	858

Predicting One Type of Volume From Another

Multiple linear regression equations were developed to predict one type of volume from another using the 1349 sawtimber trees. Equations were developed for predicting cubic-foot volume (CV) as a function of Doyle (DV), International 1/4-inch (IV), and Scribner (SV) volumes, DV as a function of CV, IV, and SV, IV as a function of CV, DV, and SV, and SV as a function of CV, DV, and IV. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in Table 6.

Predicting Board-foot Cubic-foot Ratios

Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios were calculated for each of the 1349 sawtimber trees. Average board-foot cubic-foot ratios were 4.12 (range: 2.94 to 5.80), 5.88 (range: 4.73 to 6.74), and 5.69 (range: 4.52 to 6.64) for Doyle, International 1/4-inch, and Scribner board-foot volumes, respectively.

Multiple linear regression equations were developed to predict the 3 types of board-foot cubic-foot ratios as a function of D, H, and D and H. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in Table 7.

Board-foot cubic-foot ratios for the 3 types of board-foot volumes as a function of H and D are shown in Tables 8 and 9, respectively. The values in Table 8 for a 9.6-in. top diameter limit are larger than the values in Table 8 of Fowler and Hussain (1989a) for a 7.6-in. top diameter limit, with the difference decreasing with increasing H for all 3 types of volume. The values in Table 9 for a 9.6-in. top diameter limit vary from

Table 6. Regression equations for predicting cubic-foot (CV) and Doyle (DV), International 1/4-inch (IV), and Scribner (SV) board-foot volumes from the other 3 types of volumes.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CV}=1.6151+0.2311DV-0.000132DV^2$	0.997	1.00
$\hat{CV}=0.4321+0.1695IV-0.000033IV^2$	0.999	0.59
$\hat{CV}=0.5530+0.1743SV-0.000039SV^2$	0.999	0.59
$\hat{DV}=-3.6161+3.8452CV+0.022561CV^2$	0.997	5.46
$\hat{DV}=-3.1561+0.6855IV+0.000352IV^2$	0.997	5.03
$\hat{DV}=-2.8182+0.7096SV+0.000334SV^2$	0.998	4.55
$\hat{IV}=-2.0596+5.8297CV+0.008833CV^2$	0.999	3.90
$\hat{IV}= 6.3775+1.3906DV-0.000572DV^2$	0.997	5.92
$\hat{IV}= 0.6581+1.0304SV-0.000035SV^2$	1.000	0.58
$\hat{SV}=-2.5274+5.6407CV+0.010149CV^2$	0.999	3.81
$\hat{SV}= 5.4261+1.3539DV-0.000519DV^2$	0.998	5.28
$\hat{SV}=-0.6256+0.9701IV+0.000034IV^2$	1.000	0.56

Table 7. Regression equations for predicting Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of D, H, and D and H.

Type of Ratio	Regression Equation	R^2	$s_{y \cdot x}$
Doyle	$\hat{DCR}$=1.546+0.185D	0.863	0.22
	$\hat{DCR}$=3.154+0.305H	0.766	0.29
	$\hat{DCR}$=1.952+0.127D+0.124H	0.907	0.18
International 1/4-inch	$\hat{ICR}$=4.849+0.0742D	0.476	0.23
	$\hat{ICR}$=5.382+0.157H	0.700	0.18
	$\hat{ICR}$=5.343+0.00412D+0.152H	0.700	0.18
Scribner	$\hat{SCR}$=4.447+0.0893D	0.565	0.23
	$\hat{SCR}$=5.122+0.179H	0.742	0.18
	$\hat{SCR}$=4.944+0.0188D+0.153H	0.750	0.18

Table 8. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of H.

H (sticks)	Board-foot Cubic-foot Ratio		
	DCR	ICR	SCR
1	3.5	5.5	5.3
2	3.8	5.7	5.5
3	4.1	5.9	5.7
4	4.4	6.0	5.8
5	4.7	6.2	6.0
6	5.0	6.3	6.2
7	5.3	6.5	6.4
8	5.5	6.6	6.6
9	5.9	6.8	6.7
10	6.2	7.0	6.9

Table 9. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of D.

D (inches)	Board-foot Cubic-foot Ratio		
	DCR	ICR	SCR
11	3.6	5.7	5.4
12	3.8	5.7	5.5
13	4.0	5.8	5.6
14	4.1	5.9	5.7
15	4.3	6.0	5.8
16	4.5	6.0	5.9
17	4.7	6.1	6.0
18	4.9	6.2	6.1
19	5.1	6.3	6.1
20	5.2	6.3	6.2
21	5.4	6.4	6.3
22	5.6	6.5	6.4
23	5.8	6.6	6.5
24	6.0	6.6	6.6
25	6.2	6.7	6.7

6 to 16% higher for D=11 in., 3 to 5% higher for D=15 in., 1 to 2% lower for D=20 in., and 4 to 5% lower for D=25 in. compared to the values in Table 9 of Fowler and Hussain (1989a) for a 7.6-in. top diameter limit.

Sawtimber VBAR Equations

Cubic-foot and board-foot VBAR prediction equations

A comparison of various multiple linear regression equations based on goodness-of-fit and simplicity indicated that the following prediction equations compared favorably to all other equations examined for cubic-foot and the 3 board-foot VBARs:

1. Height independent variables

$$\widehat{VBAR}=\hat{\beta}_0+\hat{\beta}_1 H+\hat{\beta}_2 \frac{1}{H}$$

2. Height and DBH independent variables

$$\widehat{VBAR}=\hat{\beta}_0+\hat{\beta}_1 H+\hat{\beta}_2 \frac{1}{H}+\hat{\beta}_3 D+\hat{\beta}_4 \frac{1}{D}$$

where $\widehat{VBAR}$ is predicted VBAR, H is MH in 100-in. sticks to a 9.6-in. top diameter limit, and D is DBH in inches. $\hat{\beta}_0$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$ are the sample regression coefficients related to the independent variables.

Table 10 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs and associated values of R^2 and $s_{y \cdot x}$ for the height independent variable models. Table 11 shows the cubic-foot and 3 board-foot VBARs for various MHs based on Equations 5-8. The values in Table 11 for a 9.6-in. minimum top diameter are, in

Table 10. Estimated intercepts ($\hat{\beta}_0$), regression coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$, and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and 3 board-foot VBAR prediction equations with independent variables based on height only.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2	$s_{y \cdot x}$
(5) Cubic-foot[a]	9.706	3.822	- 5.641	0.962	1.58
(6) Doyle[b]	5.079	25.131	- 3.046	0.971	7.57
(7) International 1/4-inch[c]	41.653	26.908	-25.239	0.960	10.72
(8) Scribner[d]	37.344	26.839	-22.756	0.962	10.21

[a] $\widehat{VBAR}=9.706+3.822 \cdot H-5.641 \cdot \frac{1}{H}$

[b] $\widehat{VBAR}=5.079+25.131 \cdot H-3.046 \cdot \frac{1}{H}$

[c] $\widehat{VBAR}=41.653+26.908 \cdot H-25.239 \cdot \frac{1}{H}$

[d] $\widehat{VBAR}=37.344+26.839 \cdot H-22.756 \cdot \frac{1}{H}$

Table 11. VBARs in cu.ft./sq.ft. or bd.ft./sq.ft. for the 4 types of volume for various values of MH.

MH (sticks)	Cu.ft./Sq.ft.	Bd.ft./Sq.ft.		
		Doyle	Scribner	Int. 1/4-inch
1	7.89	27	41	43
2	14.53	54	80	83
3	19.29	79	110	114
4	23.58	105	139	143
5	27.69	130	167	171
6	31.70	155	195	199
7	36.65	181	222	226
8	39.58	206	249	254
9	43.48	231	276	281
10	47.36	256	303	308

general, larger than the values in Table 2 of Fowler and Hussain (1989b) for a 7.6-in. minimum top diameter with the difference increasing slightly with height for cu.ft. VBARs (from -1.3 to 2.4%) and decreasing with height for Doyle (from 50.0 to 10.8%), International 1/4-inch (from 16.2 to 4.8%), and Scribner (from 20.6 to 5.2%) bd.ft. VBARs.

Table 12 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs for the height and diameter independent variable models. Note that R^2 and $s_{y \cdot x}$ for these equations are somewhat larger and smaller, respectively, than for the respective equations based on height only (Table 10). A sawtimber cubic-foot VBAR table based on Equation 9 is shown in Table 13, and Doyle, International 1/4-inch, and Scribner board-foot VBAR tables are shown in Tables 14, 15, and 16 based on Equations 10, 11, and 12, respectively.

Predicting One Type of VBAR From Another

Multiple linear regression equations were developed to predict one type of VBAR from another using the 1349 sawtimber trees. Equations were developed for predicting cubic-foot VBAR (CR) as a function of Doyle (DR), International 1/4-inch (IR), and Scribner (SR) VBARs, DR as a function of CR, IR, and SR, IR as a function of CR, DR, and SR, and SR as a function of CR, DR, and IR. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in Table 17.

Table 12. Estimated intercepts ($\hat{\beta}_0$), regression coefficients $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$ and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and 3 board-foot VBAR prediction equations with independent variables based on height and diameter.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2	$s_{y \cdot x}$
(9) Cubic-foot[a]	20.944	4.816	- 5.184	-0.844	- 38.122	0.985	1.01
(10) Doyle[b]	-34.109	25.448	- 5.605	1.104	320.632	0.971	7.54
(11) International 1/4-inch[c]	55.322	32.737	-25.861	-3.270	182.874	0.977	8.12
(12) Scribner[d]	55.843	32.035	-23.226	-2.957	152.779	0.976	8.08

[a] $\hat{VBAR}=20.944+4.816 \cdot H-5.184 \cdot \frac{1}{H}-0.844 \cdot D-38.122 \cdot \frac{1}{D}$

[b] $\hat{VBAR}=-34.109+25.448 \cdot H-5.605 \cdot \frac{1}{H}+1.104 \cdot D+320.632 \cdot \frac{1}{D}$

[c] $\hat{VBAR}=55.322+32.737 \cdot H-25.861 \cdot \frac{1}{H}-3.270 \cdot D+182.874 \cdot \frac{1}{D}$

[d] $\hat{VBAR}=50.843+32.035 \cdot H-23.226 \cdot \frac{1}{H}-2.957 \cdot D+152.779 \cdot \frac{1}{D}$

Table 13. VBAR showing cu.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 9).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	7.83	15.23	20.91	26.16	31.24	36.23			
12	7.27	14.68	20.36	25.61	30.68	35.67	40.61		
13	6.67	14.08	19.76	25.01	30.08	35.07	40.01	44.92	
14	6.04	13.45	19.12	24.37	29.45	34.44	39.38	44.28	
15	5.37	12.78	18.46	23.71	28.79	33.77	38.71	48.51	53.38
16		12.10	17.78	23.03	28.10	33.09	38.03	42.94	47.83
17		11.39	17.07	22.32	27.40	32.39	37.32	42.23	47.12
18		10.67	16.35	21.60	26.68	31.67	36.61	41.51	46.40
19		9.94	15.62	20.87	25.94	30.93	35.87	40.78	45.67
20		9.20	14.88	20.13	25.20	30.19	35.13	40.04	44.93
21			14.12	19.37	24.45	29.44	34.38	39.28	44.17
22			13.36	18.61	23.69	28.68	33.61	38.52	43.41
23			12.59	17.84	22.92	27.91	32.85	37.75	42.64
24			11.82	17.07	22.14	27.13	32.07	36.98	41.87
25			11.04	16.29	21.36	26.35	31.29	36.20	41.09

Table 14. VBAR table showing Doyle bd.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 10).

DBH	MH in Sticks							
(Inches)	1	2	3	4	5	6	7	8	9
11	27.0	55.3	81.7	107.6	133.3	158.9			
12	25.7	54.0	80.3	106.2	132.0	157.6	183.2		
13	24.8	53.0	79.4	105.3	131.0	156.7	182.2	207.8	
14	24.1	52.3	78.7	104.6	130.4	156.0	181.6	207.1	
15	23.7	51.9	78.3	104.2	129.9	155.6	181.2	206.7	232.2
16		51.7	78.0	104.0	129.7	155.3	180.9	206.5	232.0
17		51.6	78.0	103.9	129.6	155.3	180.9	206.4	231.9
18		51.7	78.1	104.0	129.7	155.3	180.9	206.5	232.0
19		51.8	78.2	104.1	129.9	155.5	181.1	206.6	232.2
20		52.1	78.5	104.4	130.1	155.8	181.3	206.9	232.4
21			78.8	104.7	130.5	156.1	181.7	207.2	232.8
22			79.2	105.1	130.9	156.5	182.1	207.6	233.2
23			79.7	105.6	131.3	157.0	182.6	208.1	233.6
24			80.2	106.1	131.9	157.5	183.1	208.6	234.2
25			80.8	106.7	132.4	158.1	183.7	209.2	234.7

Table 15. VBAR table showing International 1/4-inch bd.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 11).

DBH	MH in Sticks							
(Inches)	1	2	3	4	5	6	7	8	9
11	42.9	88.5	125.6	160.5	194.5	228.1			
12	38.2	83.9	120.9	155.8	189.8	223.4	256.8		
13	33.8	79.4	116.5	151.4	185.4	219.0	252.3	285.5	
14	29.5	75.1	112.2	147.1	181.1	214.7	248.1	281.3	
15	25.3	71.0	108.1	142.9	177.0	210.6	243.9	277.1	310.2
16		67.0	104.0	138.9	172.9	206.5	239.9	273.1	306.2
17		63.0	100.1	135.0	169.0	202.6	236.0	269.2	302.2
18		59.2	96.2	131.1	165.1	198.7	232.1	265.3	298.4
19		55.4	92.4	127.3	161.3	194.9	228.3	261.5	294.6
20		51.6	88.7	123.5	157.6	191.2	224.5	257.7	290.8
21			85.0	119.8	153.9	187.5	220.8	254.0	287.1
22			81.3	116.2	150.2	183.8	217.2	250.4	283.5
23			77.7	112.5	146.6	180.2	213.5	246.7	279.8
24			74.1	108.9	143.0	176.6	209.9	243.1	276.2
25			70.5	105.4	140.0	173.0	206.4	240.6	272.6

Table 16. VBAR table showing Scribner bd.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 12).

DBH	MH in Sticks							
(Inches)	1	2	3	4	5	6	7	8	9
11	41.0	84.7	120.6	154.5	187.7	220.5			
12	36.9	80.5	116.5	150.4	183.6	216.4	249.0		
13	33.0	76.6	112.5	146.5	179.7	212.5	245.1	277.5	
14	29.2	72.8	108.7	142.7	175.9	208.7	241.3	273.7	
15	25.5	69.1	105.0	139.0	172.2	205.0	237.6	270.0	302.4
16		65.5	101.4	135.4	168.6	201.4	234.0	266.5	298.8
17		62.0	97.9	131.9	165.1	197.9	230.5	262.9	295.3
18		58.6	94.5	128.4	161.6	194.4	227.0	259.5	291.8
19		55.2	91.1	125.0	158.2	191.0	223.6	256.1	288.4
20		51.8	87.7	121.7	154.9	187.7	220.3	252.7	285.1
21			84.4	118.4	151.6	184.4	216.9	249.4	281.8
22			81.1	115.1	148.3	181.1	213.7	246.1	278.5
23			77.8	111.8	145.0	177.8	210.4	242.9	275.2
24			74.6	108.6	141.8	174.6	207.2	239.6	272.0
25			71.4	105.4	138.6	171.4	204.0	236.4	268.8

Table 17. Regression equations for predicting cubic-foot VBARs (CR) and Doyle (DR), International 1/4-inch (IR) and Scribner (SR) board-foot VBARs from the other 3 types of VBARs.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CR}=1.4150+0.2587DR-0.000418DR^2$	0.974	1.31
$\hat{CR}=0.5743+0.1747IR-0.000092IR^2$	0.997	0.44
$\hat{CR}=0.6856+0.1807SR-0.000109SR^2$	0.996	0.51
$\hat{DR}=-5.522+3.682CR+0.041043CR^2$	0.968	7.91
$\hat{DR}=-3.583+0.666IR+0.000616IR^2$	0.981	6.11
$\hat{DR}=-3.039+0.688SR+0.000605SR^2$	0.985	5.49
$\hat{IR}=-2.593+5.583CR+0.024117CR^2$	0.997	2.98
$\hat{IR}= 4.697+1.507DR-0.001666DR^2$	0.983	6.98
$\hat{IR}= 0.667+1.035SR-0.000079SR^2$	1.000	0.73
$\hat{SR}=-3.162+5.387CR+0.026262CR^2$	0.996	3.41
$\hat{SR}= 3.887+1.457DR-0.001488DR^2$	0.986	6.19
$\hat{SR}=-0.650+0.966IR+0.000072IR^2$	1.000	0.73

Guidelines for Users

We recommend the use of Equations 1-4 for estimating individual tree sawtimber volume for a 9.6-in. minimum top diameter for red pine in Michigan. If the user wants to predict one type of volume from another, the appropriate equation in Table 6 can be used. If the user needs to estimate board-foot cubic-foot ratios for one of the types of board-foot volumes, the appropriate equation in Table 7 can be used.

The new cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBAR prediction equations using DBH and height independent variables (Table 12, Equations 9-12) were somewhat more accurate than the respective equations using only height independent variables (Table 10, Equations 5-8). However, since the differences in accuracy between the 2 sets of equations were relatively small, the simpler height only VBAR equations are more than adequate for most situations.

We recommend the use of VBAR Equations 5-8 for most cruising situations. For those situations where somewhat more accuracy is needed and DBH is already measured for some other purpose(s), Equations 9-12 may be used. If the user has access to a set of VBAR equations for just one type of volume, VBARs for other types of volume can be predicted from the appropriate equation in Table 17.

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FOREST MANAGEMENT DIVISION