## STATE OF MICHIGAN DEPARTMENT OF NATURAL RESOURCES

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# MICHIGAN DEPARTMENT OF NATURAL RESOURCES FISHERIES DIVISION 

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# Estimating Angling Effort and Catch from Michigan Roving and Access Site Angler Survey Data 

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#### Abstract

Michigan Department of Natural Resources, Fisheries Division uses roving and access site angler survey methods to collect effort and catch information from inland and Great Lakes fisheries. These surveys follow a stratified design using structured sampling within strata and data reflect angling characteristics for specific locations during specific calendar and daily periods. We present equations used for estimating angling effort, harvest by species, and catch-and-release by species from these data. Variance equations for each given estimator are also provided. Equations given provide estimates per day or over multiple-days within a time period and considerations for selecting each are presented. For each type, per day or multiple-day, equations for using access site interviews (completed-fishing trip) or roving interviews (incompleted-fishing trip) in conjunction with instantaneous or interval counting techniques are given. Methods for estimating targeted catch and targeted effort are also discussed.

Presentation of these equations has three purposes: first, to promote consistency in angler survey techniques; second, to compile existing equations which have not previously been reported and; third, to reflect current methodology. These purposes all promote comparability and accuracy of Michigan angler surveys.

Equations used in previous years are also presented. Compilation of these equations provides historical documentation and preservation of methodologies used prior to 1998.


## Introduction

Michigan Department of Natural Resources (MDNR), Fisheries Division conducts angler surveys on inland and Great Lakes waters each year to estimate angling effort and catch or harvest (usually by species; e.g., Lockwood 1996; Rakoczy and Svoboda 1995). However, only partial documentation exists detailing equations used to calculate these estimates (e.g., Schneider and Lockwood 1979; Ryckman 1981; Ryckman and Lockwood 1985; Austen et al. 1995) and this existing documentation does not reflect recent modifications (e.g., Lockwood 1997). Here we provide detailed descriptions of the equations currently used to estimate angling effort, and catch or harvest.

Michigan angler surveys consist of two separate sampling components: interviews of angling trips and counts of anglers. When a survey clerk waits at a given location, such as a boat launch site, to interview anglers as they complete their fishing trip the survey is referred to as an access site survey; when a survey clerk interviews anglers as they are actively fishing, and prior to the completion of their fishing trip, the survey is referred to as a roving survey. That is, access site surveys use completed-trip interviews and roving surveys use incompleted-trip interviews, and interviews of only one type are collected for a given time period and angling mode. Further descriptions of techniques associated with these survey types are given by Pollock et al. 1997, Pollock et al. 1994, Robson 1991, and Hayne 1991.

Interviews are records of angler or angler party fishing trip and catch information. Typically, number of anglers in the party, length of the fishing trip, catch or harvest by species, date, angling mode, and locale of fishing are recorded. Number of fishing trips taken or expected to be taken on the day of interview may also be recorded. Catch information may be recorded by individual angler or by angling party and choice is often determined by survey type, access or roving. Typically, angling information is collected by individual angler for roving surveys, not by angling party, to avoid angler party size bias (Lockwood 1997). Catch rates are calculated separately for completedor incompleted-trip interviews. The appropriate catch rate estimators are the ratio-of-means estimator for completed-trip interviews and the mean-of-ratios estimator for incompleted-trip interviews (Hoenig et al. 1997; Pollock et al. 1997; Lockwood 1997; Jones et al. 1995; and Pollock et al. 1994). Number of fish harvested as well as number caught and released may be estimated from angler surveys (e.g., Lockwood et al. 1995). Either the number harvested or the number caught and released may be used with the ratio-of-means or mean-of-ratios catch rate estimators. However, potential sources of catch rate error differ between harvested and caught-and-released fish. Typically, when an angler is interviewed the clerk records: the current time, time the angler reports the fishing trip began, number by species of any harvested fish in the angler's possession, and (in some cases) number by species of caught-and-released fish. For harvested fish, only the beginning time of the fishing trip is
potentially unobservable by the clerk. However for caught-and-released fish, both species type and number cannot be observed. Since the angler reports the time the fishing trip began, catch rates of either harvested or caught-and-released fish are subject to reporting errors (recall, rounding etc.). Catch rates of caught-and-released fish are subject to numerous additional errors. Number of fish caught-and-released are subject to recall, prestige and rounding errors, and species of fish are more likely to be misidentified.

Counts are made of individual anglers, or angling units that may represent multiple anglers, such as boats, vehicles or ice shanties. Two types of counts are utilized: interval and instantaneous. For interval counts, boats (or other units) are counted as they enter a fishing area (Fabrizio et al. 1991). Interval counting technique is used primarily when boats enter a fishery and then disappear from the survey clerk's view. For example, at a Great Lakes port anglers may travel several miles out onto the lake to fish. Here, boats are counted as they leave the port and enter the lake. Interval counts may be 15 minutes or more in duration. When interval counts are made, the clerk will not have an opportunity to interview anglers until they return and have completed their fishing trip. Thus, completed-trip interviews will be collected and angling party information (length of trip and number of anglers in each party) recorded.

Instantaneous counts are counts of boats or anglers etc., visible during a given short period (instant). When instantaneous counts are made of units which may represent more than one angler, number of anglers in each party is required, and typically completed-trip interviews by angling party are collected. In the case of ice shanties, counts may be of occupied shanties or all shanties present (occupied and unoccupied). When all shanties present are counted, a separate survey is conducted to estimate the proportion of occupied shanties. Then, estimated occupied shanties for a given site and time period are the product of the mean count of all shanties and proportion of occupied shanties. For instantaneous counts of anglers, completed-trip or incompleted-trip interviews may be collected.

Single or multiple interval or instantaneous counts may be made in a sampling day. We give instantaneous count equations for boats and anglers, while only boats are noted for interval equations since other count units are rarely enumerated. However, substitution with units other than boats or anglers is possible for either counting type. Likewise, equations noted for use with boat interviews also apply to any angling mode where the unit may represent more than one angler.

Angling effort is reported as estimated angler-hours or estimated angler-trips. Angler-hours reflect total hours involved in fishing by anglers for a given time period at a given location. These fishing hours cover the time from arrival at a site to departure from that site and include within site travel times such as the time it takes an angler to walk from their parked vehicle to a chosen location within a site to fish. Angler trips reflect only the number of times anglers fish at a given location. An angler may, for example, fish several hours early in the day, stop fishing and leave the site, and
then return later in the day and fish several more hours. While this is a single day, multiple hours have been fished and two angler trips have been made.

From the sample of counts and interviews, catch rate $(R)$ and angling effort $(E)$ are initially calculated and then catch $(C)$ is estimated as their product. Since catch rate is measured in catch per hour, angler-hours are the effort measurement $(E)$ used for expansion of catch rate $(R)$ to estimate catch $(C)$. Angling effort, catch rates and ultimately estimated catch are usually determined by site, angling mode (boat, shore etc.) and time period. Time periods may be individual days (daily estimates) or multiple days (multiple-day estimates), such as all week days within a month. The equations that are presented deal with these two major categories of estimates, "daily estimates" and "multiple-day estimates". For daily estimates, catch rate, angling effort and catch are estimated for each day sampled. While for multiple-day estimates, daily catch-rate information is pooled and counts are averaged across days with catch as their product. Either daily or multiple-day estimates and their variances are summed over time periods and angling modes to provide a total estimate.

When counting units contain both fishing and non-fishing activity (such as fishing and nonfishing boats), appropriate interview data must be collected to separate estimated fishing effort from estimated non-fishing effort. For example, counts of boats may indiscriminately include both fishing and non-fishing boats. In this situation, both fishing and non-fishing boating parties are interviewed and the activity type recorded. Thus, the composition of boating activity types in the interview data set is assumed to reflect that of the boating population at that site. Regardless of method, daily or multiple day, a single ratio to adjust for non-fishing boats is estimated for a multiple-day period. When daily estimation methods are used, this adjustment ratio and its variance are applied to effort and harvest estimates after they have been combined over multiple-day periods. When multiple-day estimation methods are used, this adjustment ratio and its variance are applied to effort only, prior to calculation of harvest estimates.

MDNR, Fisheries Division maintains a survey methods manual (Schneider et al. 1981) describing (among other things) how creel survey estimates are made (e.g., Ryckman 1981). Many documents cite this as a source for detail on how these estimates are made. Unfortunately, this "living document" gets updated as the methods are updated, and documentation on historical methodology could be lost. In addition, the survey methods manual only contains partial documentation of estimation equations used. Additional equations are found in Ryckman and Lockwood (1985), and Schneider and Lockwood (1979). To help alleviate these problems, we have included (in Appendix 1) the estimation equations used prior to our revision. It is highly likely that creel survey estimates in documents published prior to 1998 that reference the survey methods manual used the equations contained in Appendix 1, since this set of equations had remained substantially unchanged since the 1960's.

At the time this report was prepared, daily estimates were being implemented for Great Lakes angler surveys, and multiple-day estimates were being implemented for most inland angler surveys. While ongoing and future estimation of catch and harvest from angler surveys will follow the methods described here, previously published estimates of catch and harvest based on MDNR, Fisheries Division angler surveys [specifically those referencing Ryckman (1981)] used the multipleday methods (usually days of a day type within a month) found in Appendix 1, not the methods we recommend below for calculating catch rates and estimates of variances.

The next section presents issues to consider when choosing between daily estimates and multipleday estimates, followed by sections describing the equations used for the two methods. Symbols and subscripts used throughout this report are given in Table 1.

## Choosing Between Daily and Multiple-Day Estimates

The difference between the daily and multiple-day (usually all days of a type [either week days or weekends and holidays]) estimates rests in how interviews are treated. For the daily estimates, catch rates are calculated for each day based on the interviews and these catch rates are multiplied by effort for that day to provide estimates of catch for that day. Variances are calculated for catch rate, effort, and catch for each day that sampling occurred, and this explicitly takes into account any differences in catch rates among days. Multiple-day estimates treat all interviews within a longer period as though they were random samples from that longer time period. A single catch rate is calculated for the longer period and is multiplied by effort for the longer time period to produce estimates of catch for that period. This approach ignores day-to-day differences in catch rates.

Potential problems can arise. First, if actual differences in catch rates exist among days this would tend to cause a multiple-day estimate to underestimate the variance associated with the catch rate for the time period (by ignoring cluster or day effects). Second, if there is a correlation (either negative or positive) between daily catch rates and daily effort within a longer time period, pooling data over the longer time period can introduce a bias in estimates of catch (see Austen et al. 1995 for further discussion). On the other hand, it can be quite difficult to obtain adequate sample sizes for a single day. Estimated variance for ratio estimates, such as catch rates, become biased when sample sizes are small and estimates and the variances cannot be calculated if no or one interview are all that is obtained on a given day. Excluding these days has the potential to introduce bias because these are likely to also be days when effort was low.

When large numbers of interviews are regularly collected for each day of sampling, the clear choice is to use daily estimates. When most days contribute only one or a few interviews, day or
cluster effects are less important and daily estimates are difficult to calculate. In these cases multipleday periods become the only option. In other cases the number of daily samples are intermediate and the choice between daily and multiple-day periods can be more difficult. In these cases it is worth exploring the frequency distribution of interviews per day and relationships between daily catch rate and daily effort.

Although we have noted important considerations and an approach to data analysis, we have not specified a set of criteria that will unambiguously determine whether a daily or multiple-day estimate should be used. Currently multiple-day estimates are used in inland surveys and daily estimates for Great Lakes surveys because of general differences in data characteristics. It is possible that improved estimates would be obtained if different estimators were used for different sites on the same lake or even for different time periods at the same site. It is clear that there is much to know about how these different estimators respond to data characteristics, and research including simulations and trials of different estimators on the same data could be helpful in improving how estimators are chosen.

## Targeted Effort and Associated Quantities

The bulk of the estimation section that follows assumes that all fishing effort and catch or harvest obtained by that effort is the variable of interest. In some cases we are more interested in effort directed at certain targets. For example we may want a measure of effort that we believe is proportional to fishing mortality on yellow perch Perca flavescens. Total effort could be misleading because it varies over time with changes in effort directed at salmon and trout. Likewise, we may want to use catch per unit effort (CPUE) of, say, chinook salmon Oncorhynchus tshawytscha as an index of that species abundance. Using the ratio of catch of that species to all effort as CPUE could be misleading since this ratio would go down just because effort directed toward yellow perch increased.

As part of the interview process, information may be collected on what targets are being sought. This information can be used to estimate both targeted effort and catch or harvest associated with that effort. In this document we include a subsection at the end of the estimation methods section describing the general algorithm currently being used to estimate targeted effort, and associated catch and harvest, by using information on targets sought reported during the interview process. There are a variety of ways to define categories or targets for analysis. The targets listed on the interview forms may be overlapping (e.g., walleye Stizostedion vitreum and yellow perch, walleye, yellow perch) and some targets get very little effort and would reasonably be combined with other targets. The presence
of distinct fisheries differs among locations. For example, at some Great Lakes ports there are clearly distinct groups of salmon and yellow perch anglers using different gear and fishing in different locations. In some inland lakes most anglers are using similar strategies and are targeting the same suite of species. Hence, definition of appropriate and reasonable targets can depend upon the situation and the question at hand. We do not attempt to provide a "standard" set of defined groups.

## Estimation Methods

Equations that follow are for a given fishing mode (e.g., boat, pier) and day type (e.g., weekday or weekend and holiday). Subscripts for these other variables have been dropped for simplicity. Also, we assume that interviews are predominantly completed- or incompleted-trip by design and that interviews of only one type are used in analyses. Methods used in the calculation of catch rates depend upon what type of interview is being used. Effort estimates are based primarily upon counts of anglers or groups of anglers and counts can be either instantaneous or over time intervals. Methods used to estimate effort depend upon both the counting unit (single angler or groups) and whether counts are instantaneous or over-time intervals.

## Daily Estimates

For these daily estimates, catch rates are calculated for individual days and multiplied by effort for that day to estimate the daily harvest. In some cases catch rates with associated variances cannot be calculated for a given day because no or only one fishing interview is obtained. In those cases, catch rates need to be "borrowed" by using the mean over a longer time period. The longer time period is usually all sampled days (for the same day type and mode) within the month, and in our descriptions below we will describe this mean as the "monthly mean".

Catch rate estimators.-Recall that anglers may be interviewed upon completion of their fishing trip (completed-trip interviews) or while actively fishing (incompleted-trip interviews). Catch rates and corresponding variances for these two types of interviews are calculated differently.

Completed-trip interviews.-Estimated catch per angler-hour for each day $d$ is calculated as the ratio of means:

$$
\begin{equation*}
\hat{R}_{d}=\frac{\sum_{i=1}^{k_{d}} c_{d i}}{\sum_{i=1}^{k_{d}} h_{d i}}=\frac{\sum_{i=1}^{k_{d}} c_{d i} / k_{d}}{\sum_{i=1}^{k_{d}} h_{d i}}=\frac{\overline{c_{d}}}{\overline{h_{d}}}, \tag{1}
\end{equation*}
$$

where,

$$
\begin{aligned}
& c_{d i}=\text { total catch of a particular species by angler (or angler party) } i \text { on day } \mathrm{d}, \\
& h_{d i}=\text { total hours fished by angler (or angler party) } \mathrm{i} \text { on day } \mathrm{d}, \\
& k_{d}=\text { total number of anglers (or angler parties) interviewed on day } \mathrm{d} .
\end{aligned}
$$

Total hours fished by angler or angler party ( $h$ ) represent the total recorded hours fished for a given interview ( $h=a^{*} t$ ) for anglers in a fishing party ( $a$ ) and trip length for that fishing party $(t)$. The estimated variance (Cochran 1977) of catch per angler-hour (1) is:

$$
\begin{equation*}
\operatorname{Vâr}\left(\hat{R}_{d}\right)=\left[\frac{1}{\sqrt{k_{d}}\left(\overline{h_{d}}\right)} \sqrt{\frac{\left[\sum_{i=1}^{k_{d}} c_{d i}^{2}-2 \hat{R}_{d}\left(\sum_{i=1}^{k_{d}} c_{d i} h_{d i}\right)+\hat{R}_{d}^{2}\left(\sum_{i=1}^{k_{d}} h_{d i}^{2}\right)\right]}{k_{d}-1}}\right] . \tag{2}
\end{equation*}
$$

Incompleted-trip interviews.-Estimated catch per angler-hour for day $d$ is calculated as the mean of ratios:

$$
\begin{equation*}
\bar{R}_{d}=\frac{\sum_{i=1}^{k_{d}}\left(c_{d i} / h_{d i}\right)}{k_{d}}, \tag{3}
\end{equation*}
$$

and estimated variance of mean-of-ratios catch per angler-hour:

$$
\begin{equation*}
\operatorname{Vâr}\left(\bar{R}_{d}\right)=\frac{\sum_{i=1}^{k_{d}}\left(c_{d i} / h_{d i}\right)^{2}-\frac{\left(\sum_{i=1}^{k_{d}}\left(c_{d i} / h_{d i}\right)\right)^{2}}{k_{d}}}{k_{d}\left(k_{d}-1\right)} . \tag{4}
\end{equation*}
$$

Catch rates for sampled days with no interviews.-In some cases even though sampling occurs, no or only one fishing interview is obtained. In these cases either no catch rate can be estimated or the catch rate is unreliable and has no associated estimate of variance. To deal with this situation we use
monthly mean catch rates in place of specific daily catch rates in calculations. For completed-trip interviews, the monthly mean catch rate from (1) is used:

$$
\begin{equation*}
\tilde{\hat{R}}_{p}=\frac{\sum_{d=1}^{w} \hat{R}_{d}}{w} \tag{5}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \mathrm{w}=\text { number of sampled days in stratum with fishing interviews, } \\
& \mathrm{p}=\text { time period }
\end{aligned}
$$

and estimated variance is:

$$
\begin{equation*}
\operatorname{Var}\left(\tilde{\hat{R}}_{p}\right)=\frac{\sum_{d=1}^{w} \hat{R}_{d}^{2}-\frac{\left(\sum_{d=1}^{w} \hat{R}_{d}\right)^{2}}{w}}{(w-1)} . \tag{6}
\end{equation*}
$$

Note that equation (6) is not a variance estimate of the mean for a strata, rather it is the variance estimate for "some" day in the stratum. Thus, $\tilde{\hat{R}}_{p}$ and $\operatorname{Var}\left(\tilde{\hat{R}}_{p}\right)$ replace missing values of $\hat{R}_{d}$ and $\operatorname{Var}\left(\hat{R}_{d}\right)$ for that stratum ("month").

Similar to missing catch data for completed-trip interviews, on sample days where an inadequate amount of incompleted-trip catch information was collected, missing values are replaced by:

$$
\begin{equation*}
\tilde{\bar{R}}_{p}=\frac{\sum_{d=1}^{w} \bar{R}_{d}}{w}, \tag{7}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\tilde{\bar{R}}_{p}\right)=\frac{\sum_{d=1}^{w} \bar{R}_{d}^{2}-\frac{\left(\sum_{d=1}^{w} \bar{R}_{d}\right)^{2}}{w}}{(w-1)} \tag{8}
\end{equation*}
$$

Angling effort.-Effort data are collected in the form of interval or instantaneous counts depending upon logistics, and different calculations are needed depending upon the type of data available. First we present effort calculations assuming that all counts are of angling units. After we combine daily effort for multiple-day periods, we apply an adjustment ratio to correct for non-fishing units.

Interval counts.-Interval counts are usually applied to the number of boats passing a fixed point (e.g., leaving a harbor) and all our applications of interval counts are for groups of anglers. In what follows we use the term "boat" to refer to the unit that is counted although it is conceivable that other units might be counted.

Estimated number of boats on day $d$ from interval count $j$ is calculated as:

$$
\begin{equation*}
B_{d j}=F_{d} \frac{b_{d j}}{L_{d j}} \tag{9}
\end{equation*}
$$

where,

$$
\begin{aligned}
F_{d} & =\text { number of fishable hours on day } \mathrm{d}, \\
b_{d j} & =\text { boats or other units counted on day } \mathrm{d} \text { during interval } \mathrm{j}, \\
L_{d j} & =\text { duration (hours) of interval count } \mathrm{j} \text { on day } \mathrm{d} .
\end{aligned}
$$

Here, estimated mean number of boats for day $d$ is:

$$
\begin{equation*}
\bar{B}_{d}=\frac{\sum_{j=1}^{n_{d}} B_{d j}}{n_{d}}, \tag{10}
\end{equation*}
$$

where,

$$
n_{d}=\text { number of counts made on day } \mathrm{d} .
$$

Estimated variance of mean number of boats on day $d$ is:

$$
\begin{equation*}
\operatorname{Vâr}\left(\bar{B}_{d}\right)=\frac{\sum_{j=1}^{n_{d}} B_{d j}^{2}-\frac{\left(\sum_{j=1}^{n_{d}} B_{d j}\right)^{2}}{n_{d}}}{n_{d}\left(n_{d}-1\right)} . \tag{11}
\end{equation*}
$$

Note that equation (11) and similar variance equations involving $n_{d}$ cannot be calculated when $n_{d}<2$.

From the interviews of anglers (completed-trip), mean boat angler-hours per angler trip on day $d$ is calculated as:

$$
\begin{equation*}
\bar{e}_{d}=\frac{\sum_{i=1}^{k_{d}} h_{d i}}{k_{d}} \tag{12}
\end{equation*}
$$

where,

$$
k_{d}=\text { number of angler parties interviewed, }
$$

and estimated variance of mean boat angler-hours per angler trip is:

$$
\begin{equation*}
\operatorname{Vâ} r\left(\bar{e}_{d}\right)=\frac{\sum_{i=1}^{k_{d}} h_{d i}^{2}-\frac{\left(\sum_{i=1}^{k_{d}} h_{d i}\right)^{2}}{k_{d}}}{k_{d}\left(k_{d}-1\right)} . \tag{13}
\end{equation*}
$$

Estimated angler-hours for day $d$ is:

$$
\begin{equation*}
E_{d}=\bar{B}_{d} \bar{e}_{d}, \tag{14}
\end{equation*}
$$

with estimated variance (Goodman 1960):

$$
\begin{equation*}
\operatorname{Var}\left(E_{d}\right)=\bar{B}_{d}^{2} \operatorname{Var}\left(\bar{e}_{d}\right)+\bar{e}_{d}^{2} \operatorname{Var}\left(\bar{B}_{d}\right)-\operatorname{Var}\left(\bar{e}_{d}\right) \operatorname{Var}\left(\bar{B}_{d}\right) . \tag{15}
\end{equation*}
$$

Note that equation (15) cannot be calculated when $n_{d}<2$.
On sample days when anglers were counted but an inadequate number of angler interviews were collected, substitute $\tilde{\bar{e}}_{p}$ for $\bar{e}_{d}$ in equations (14) and (15), and $\operatorname{Vâr}\left(\tilde{\bar{e}}_{p}\right)$ for $\operatorname{Var}\left(\overline{\bar{e}}_{d}\right)$ in equation (15). Here:

$$
\begin{equation*}
\tilde{\bar{e}}_{p}=\frac{\sum_{d=1}^{w} \bar{e}_{d}}{w}, \tag{16}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\tilde{\bar{e}}_{p}\right)=\frac{\sum_{d=1}^{w} \bar{e}_{d}^{2}-\frac{\left(\sum_{d=1}^{w} \bar{e}_{d}\right)^{2}}{w}}{(w-1)} \tag{17}
\end{equation*}
$$

Instantaneous boat counts.-Procedures for boats also apply to instantaneous counts for any unit consisting of multiple anglers and can be boats, trailers or other units such as occupied fishing shanties. Estimated number of boat-hours on day $d$ from instantaneous count $j$ is calculated as:

$$
\begin{equation*}
\beta_{d j}=F_{d} b_{d j}, \tag{18}
\end{equation*}
$$

where,

$$
\begin{aligned}
& F_{d}=\text { number of fishable hours on day } \mathrm{d}, \\
& b_{d j}=\text { boats or other units counted on day } \mathrm{d} \text { during count } \mathrm{j} .
\end{aligned}
$$

Estimated mean number of boat-hours or other units for day $d$ is:

$$
\begin{equation*}
\bar{\beta}_{d}=\frac{\sum_{j=1}^{n_{d}} \beta_{d j}}{n_{d}}, \tag{19}
\end{equation*}
$$

and estimated variance of mean number of boat-hours or other units on day $d$ is:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{\beta}_{d}\right)=\frac{\sum_{j=1}^{n_{d}} \beta_{d j}^{2}-\frac{\left(\sum_{j=1}^{n_{d}} \beta_{d j}\right)^{2}}{n_{d}}}{n_{d}\left(n_{d}-1\right)} \tag{20}
\end{equation*}
$$

Mean anglers per boat on day $d$ are calculated from the interviews as:

$$
\begin{equation*}
\bar{a}_{d}=\frac{\sum_{i=1}^{k_{d}} a_{d i}}{k_{d}}, \tag{21}
\end{equation*}
$$

and the estimated variance of mean anglers per boat is:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{a}_{d}\right)=\frac{\sum_{i=1}^{k_{d}} a_{d i}^{2}-\frac{\left(\sum_{i=1}^{k_{d}} a_{d i}\right)^{2}}{k_{d}}}{k_{d}\left(k_{d}-1\right)} . \tag{22}
\end{equation*}
$$

Estimated angler-hours from instantaneous counts for day $d$ is:

$$
\begin{equation*}
E_{d}=\bar{\beta}_{d} \bar{a}_{d}, \tag{23}
\end{equation*}
$$

and variance of estimated angler-hours for day $d$ is:

$$
\begin{equation*}
\operatorname{Var}\left(E_{d}\right)=\bar{\beta}_{d}^{2} \operatorname{Vâr}\left(\bar{a}_{d}\right)+\bar{a}_{d}^{2} \operatorname{Var}\left(\bar{\beta}_{d}\right)-\operatorname{Var}\left(\bar{a}_{d}\right) \operatorname{Var}\left(\bar{\beta}_{d}\right) . \tag{24}
\end{equation*}
$$

Note that equation (24) cannot be calculated when $n_{d}<2$.
On sample days when anglers were counted but an inadequate number of angler interviews were collected, substitute $\tilde{\bar{a}}_{p}$ for $\bar{a}_{d}$ in equations (23) and (24), and $\operatorname{Var}\left(\tilde{\bar{a}}_{p}\right)$ for $\operatorname{Var}\left(\bar{a}_{d}\right)$ in equation (24), where:

$$
\begin{equation*}
\tilde{\bar{a}}_{p}=\frac{\sum_{d=1}^{w} \bar{a}_{d}}{w}, \tag{25}
\end{equation*}
$$

and:

$$
\begin{equation*}
\operatorname{Var}\left(\tilde{\bar{a}}_{p}\right)=\frac{\sum_{d=1}^{w} \bar{a}_{d}^{2}-\frac{\left(\sum_{d=1}^{w} \bar{a}_{d}\right)^{2}}{w}}{(w-1)} . \tag{26}
\end{equation*}
$$

Instantaneous angler counts.-Estimated number of angler-hours on day drom instantaneous count $j$ is calculated as:

$$
\begin{equation*}
E_{d j}=F_{d} A_{d j}, \tag{27}
\end{equation*}
$$

where,

$$
\begin{aligned}
F_{d} & =\text { number of fishable hours on day } \mathrm{d}, \\
A_{d j} & =\text { anglers counted on day } \mathrm{d} \text { on count } \mathrm{j} .
\end{aligned}
$$

Estimated total angler-hours on day $d$ is:

$$
\begin{equation*}
E_{d}=\frac{\sum_{j=1}^{n_{d}} E_{d j}}{n_{d}}, \tag{28}
\end{equation*}
$$

and estimated variance of total angler-hours on day $d$ is:

$$
\begin{equation*}
\operatorname{Var}\left(E_{d}\right)=\frac{\sum_{j=1}^{n_{d}} E_{d j}^{2}-\frac{\left(\sum_{j=1}^{n_{d}} E_{d j}\right)^{2}}{n_{d}}}{n_{d}\left(n_{d}-1\right)} . \tag{29}
\end{equation*}
$$

Note that equation (29) cannot be calculated when $n_{d}<2$.

Combining daily effort estimates for multiple-day periods.-Estimated angler-hours (by day type and angling mode) during multiple-day period $p$ is:

$$
\begin{equation*}
E_{p}=\frac{D_{p}}{m_{p}} \sum_{d=1}^{m_{p}} E_{d} \tag{30}
\end{equation*}
$$

where,

$$
\begin{aligned}
D & =\text { number of days by daytype in period } p \\
m_{p} & =\text { number of sampled days in period } p \text { for which valid estimates can be made. }
\end{aligned}
$$

Recall that if $n_{d}=1$ on a day no variance can be calculated, and when $n_{d}=0$ neither $E_{d}$ nor its variance can be calculated. In the situation where $n_{d}$ is typically $>1$, these days are dropped when estimating effort and harvest for the period. When $n_{d}$ is typically equal to 1 , only days with no counts are dropped, but variances on daily harvest and effort cannot be calculated. When 1 count per sample day is made, estimated variance for the period is:

$$
\begin{equation*}
\operatorname{Vâ} r\left(E_{p}\right)=\left(\frac{D_{p}}{m_{p}}\right)^{2}\left[\frac{\sum_{d=1}^{m_{p}} E_{d}^{2}-\frac{\left(\sum_{d=1}^{m_{p}} E_{d}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)}\right] . \tag{31}
\end{equation*}
$$

When multiple counts per sample day are made:

$$
\begin{equation*}
\operatorname{Vâr}\left(E_{p}\right)=\frac{D_{p}^{2}}{m_{p}}\left(1-\frac{m_{p}}{D_{p}}\right)\left(\frac{\sum_{d=1}^{m_{p}} E_{d}^{2}-\frac{\left(\sum_{d=1}^{m_{p}} E_{d}\right)^{2}}{m_{p}}}{\left(m_{p}-1\right)}\right)+\frac{D_{p}}{m_{p}} \sum_{d=1}^{m_{p}} \operatorname{Vâr}\left(E_{d}\right) \tag{32}
\end{equation*}
$$

When angling units, such as angling boats, are not consistently distinguished from non-angling units, all units are counted and both angling and non-angling parties are interviewed. A single adjustment ratio is estimated for a multiple-day period $p$ (e.g., week days within a month):

$$
\begin{equation*}
f_{p}=\frac{k_{p}}{U_{p}}, \tag{33}
\end{equation*}
$$

where,

$$
\begin{aligned}
& k_{p}=\text { total number of angling parties interviewed in time period } p, \\
& U_{p}=\text { total number of angling and non-angling parties interviewed in time period } p,
\end{aligned}
$$

and the estimated variance is:

$$
\begin{equation*}
\operatorname{Vâ} r\left(f_{p}\right)=\frac{f_{p}\left(1-f_{p}\right)}{U_{p}} . \tag{34}
\end{equation*}
$$

This adjustment ratio and its variance are applied to effort and harvest estimates after they have been combined over multiple-day periods. Estimated angler-hours for period $p$, adjusted for non-fishing units, is:

$$
\begin{equation*}
\hat{E}_{p}=E_{p} f_{p} \tag{35}
\end{equation*}
$$

and its associated variance is:

$$
\begin{equation*}
\operatorname{Va} r\left(\hat{E}_{p}\right)=E_{p}^{2} \operatorname{Var} r\left(f_{p}\right)+f_{p}^{2} \operatorname{Var} r\left(E_{p}^{2}\right)-\operatorname{Va} r\left(f_{p}\right) \operatorname{Va} r\left(E_{p}\right) . \tag{36}
\end{equation*}
$$

Estimated angler trips.-Length of angling trip is assumed to be relatively constant over days and estimated angler trips are estimated by period only. Estimated angler trips are the quotient of estimated angler-hours and mean length of angler trip. Mean length of angler trip, from completedtrip interviews, is:

$$
\begin{equation*}
\bar{t}_{p}=\frac{\sum_{i=1}^{k_{p}} t_{p i}}{k_{p}} \tag{37}
\end{equation*}
$$

where,

$$
k_{p}=\text { number of anglers interviewed in period } p
$$

Estimated variance of angler trip length is:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{t}_{p}\right)=\frac{\sum_{i=1}^{k_{p}} t_{p i}^{2}-\frac{\left(\sum_{i=1}^{k_{p}} t_{p i}\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{38}
\end{equation*}
$$

Estimated angler trips would be:

$$
\begin{equation*}
\hat{\varepsilon}_{p}=\frac{\hat{E}_{p}}{\bar{t}_{p}} \tag{39}
\end{equation*}
$$

and estimated variance, for independent estimates (derived from Freese 1962), of estimated angler trips would be calculated as:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{\varepsilon}_{p}\right)=\hat{\varepsilon}_{p}^{2}\left[\frac{\operatorname{Var}\left(\hat{E}_{p}\right)}{\hat{E}_{p}^{2}}+\frac{\operatorname{Var}\left(\bar{t}_{p}\right)}{\bar{t}_{p}^{2}}\right] \tag{40}
\end{equation*}
$$

Note that $E_{p}$ and $\operatorname{Var}\left(E_{p}\right)$ are substituted for $\hat{E}_{p}$ and $\operatorname{Var}\left(\hat{E}_{p}\right)$ when only fishing units are counted.

Estimated angler days.-Estimated angler days are the quotient of estimated angler trips and estimated mean number of angler trips per interview per day. Mean number of angler trips per interview is assumed to be relatively consistent over days within a sample period, and is estimated by period only. Mean number of angler trips, from incompleted- or completed-trip interviews, is:

$$
\begin{equation*}
\bar{T}_{p}=\frac{\sum_{i=1}^{k_{p}} T_{p i}}{k_{p}} \tag{41}
\end{equation*}
$$

where,

$$
T=\text { reported number of fishing trips taken per day during period } p \text { by angler } i .
$$

Estimated variance of mean number of angler trips is:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{T}_{p}\right)=\frac{\sum_{i=1}^{k_{p}} T_{p i}^{2}-\frac{\left(\sum_{i=1}^{k_{p}} T_{p i}\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{42}
\end{equation*}
$$

Estimated angler days would be:

$$
\begin{equation*}
\hat{S}_{p}=\frac{\hat{\varepsilon}_{p}}{\bar{T}_{p}} \tag{43}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{S}_{p}\right)=\hat{S}_{p}^{2}\left[\frac{\operatorname{Var}\left(\hat{\varepsilon}_{p}\right)}{\hat{\varepsilon}_{p}^{2}}+\frac{\operatorname{Va} r\left(\bar{T}_{p}\right)}{\bar{T}_{p}^{2}}\right] \text {, } \tag{44}
\end{equation*}
$$

which assumes $\hat{\varepsilon}_{p}$ and $\bar{T}_{p}$ are independent.
Estimated Catch or Harvest.-Estimated catch or harvest is the product of catch rate ( $R$ ) and angling effort $(E)$. Estimates are calculated separately for each angling mode and then typically summed to provide total estimates.

Daily.-Estimated catch rate is used the same way in the calculation of harvest whether it is based on the incompleted- or completed-trip estimator. Hence we use $R_{d}$ for catch rate here, which represents either $\hat{R}_{d}$ or $\bar{R}_{d}$ as appropriate. Note that daily estimates for catch use the effort estimate unadjusted for non-fishing units. Estimated daily catch or harvest is:

$$
\begin{equation*}
C_{d}=E_{d} R_{d}, \tag{45}
\end{equation*}
$$

and its estimated variance is:

$$
\begin{equation*}
\operatorname{Vâ} r\left(C_{d}\right)=R_{d}^{2} \operatorname{Var}\left(E_{d}\right)+E_{d}^{2} \operatorname{Var}\left(R_{d}\right)-\operatorname{Vâr}\left(E_{d}\right) \operatorname{Vâ} r\left(R_{d}\right) . \tag{46}
\end{equation*}
$$

Multiple-day period.-Estimated catch or harvest by day type, angling mode and species during multiple-day period $p$ is:

$$
\begin{equation*}
C_{p}=\frac{D_{p}}{m_{p}} \sum_{d=1}^{m_{p}} C_{d}, \tag{47}
\end{equation*}
$$

and variance of catch or harvest is:

$$
\begin{equation*}
\operatorname{Va} \hat{a}\left(C_{p}\right)=\frac{D_{p}^{2}}{m_{p}}\left(1-\frac{m_{p}}{D_{p}}\right)\left(\frac{\sum_{d=1}^{m_{p}} C_{d}^{2}-\frac{\left(\sum_{d=1}^{m_{p}} C_{d}\right)^{2}}{m_{p}}}{\left(m_{p}-1\right)}\right)+\frac{D_{p}}{m_{p}} \sum_{d=1}^{m_{p}} \operatorname{Va} r\left(C_{d}\right) \tag{48}
\end{equation*}
$$

Estimated catch or harvest, adjusted for non-fishing units, is:

$$
\begin{equation*}
\hat{C}_{p}=C_{p} f_{p}, \tag{49}
\end{equation*}
$$

and its variance is:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{C}_{p}\right)=C_{p}^{2} \operatorname{Va} r\left(f_{p}\right)+f_{p}^{2} \operatorname{Var}\left(C_{p}^{2}\right)-\operatorname{Var}\left(f_{p}\right) \operatorname{Var}\left(C_{p}\right) . \tag{50}
\end{equation*}
$$

## Multiple-Day Estimates

Equations similar to those used for daily estimates are used for multiple-day estimates. To ensure that the reader acquires a clear picture of methods used in Michigan angler surveys, some redundancy will occur.

Catch rate estimators.-Recall from previous text that anglers may be interviewed upon completion of their fishing trip (completed-trip interviews) or while actively fishing (incompleted-trip interviews). Catch rates and corresponding variances for these two types of interviews are calculated differently.

Completed-trip interviews.-Estimated catch per angler-hour from completed-trip interviews for period $p$ is calculated as:

$$
\begin{equation*}
\hat{R}_{p}=\frac{\sum_{i=1}^{k_{p}} c_{p i}}{\sum_{i=1}^{k_{p}} h_{p i}}=\frac{\sum_{i=1}^{k_{p}} c_{p i} / k_{p}}{\sum_{i=1}^{k_{p}} h_{p i} / k_{p}}=\frac{\overline{c_{p}}}{\overline{h_{p}}}, \tag{51}
\end{equation*}
$$

where,

$$
c_{p i}=\text { total catch of a particular species by angler (or angler party) } i,
$$

$$
\begin{aligned}
& h_{p i}=\text { total angler hours fished by angler (or angler party) } i, \\
& k_{p}=\text { total number of anglers (or angler parties) interviewed, }
\end{aligned}
$$

and the estimated variance (Cochran 1977) of catch per angler-hour for completed-trip interviews for period $p$ is calculated as:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{R}_{p}\right)=\left[\frac{1}{\sqrt{k_{p}}\left(\overline{h_{p}}\right)} \sqrt{\frac{\left[\sum_{i=1}^{k_{p}} c_{p i}^{2}-2 \hat{R}_{p}\left(\sum_{i=1}^{k_{p}} c_{p i} h_{p i}\right)+\hat{R}_{p}^{2}\left(\sum_{i=1}^{k_{p}} h_{p i}^{2}\right)\right]}{k_{p}-1}}\right] . \tag{52}
\end{equation*}
$$

Incompleted-trip interviews.-Estimated catch per angler-hour from incompleted-trip interviews for period $p$ is calculated as:

$$
\begin{equation*}
\bar{R}_{p}=\frac{\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)}{k_{p}} \tag{53}
\end{equation*}
$$

and estimated variance of mean-of-ratios catch per angler-hour:

$$
\begin{equation*}
\operatorname{Vâ} r\left(\bar{R}_{p}\right)=\frac{\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)^{2}-\frac{\left(\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{54}
\end{equation*}
$$

Angling effort.-Recall that effort data are collected in the form of interval or instantaneous counts depending upon logistics and that single or multiple counts may be taken each sampling day. Only one form of count is used within a time period and generally the number of counts taken each day is consistent.

Interval counts.-Estimated number of boats for period p from interval count j on day d is:

$$
\begin{equation*}
B_{p d j}=F_{p} \frac{b_{d j}}{L_{d j}}, \tag{55}
\end{equation*}
$$

where,

$$
\begin{aligned}
& F_{p}=F_{d} * D, \\
& b_{d j}=\text { boats or other units counted on day } d \text { during interval } j,
\end{aligned}
$$

$$
L_{d j}=\text { duration (hours) of interval count } j \text { on day } d \text {. }
$$

When $n_{d}>1$, estimated mean number of boats for period $p$ based on day $d$ is:

$$
\begin{equation*}
\bar{B}_{p d}=\frac{\sum_{j=1}^{n_{d}} B_{p d j}}{n_{d}}, \tag{56}
\end{equation*}
$$

and the variance is estimated from (11). Estimated mean number of boats for period $p$ is calculated as:

$$
\begin{equation*}
\bar{B}_{p}=\frac{\sum_{d=1}^{m_{p}} \bar{B}_{p d}}{m_{p}} \text {, } \tag{57}
\end{equation*}
$$

where,

$$
m_{p}=\text { number of sampled days in which counts were made. }
$$

When one count per sample day is made ( $n_{d}=1$ ), the estimated variance is:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{B}_{p}\right)=\frac{\sum_{d=1}^{m_{p}} \bar{B}_{p d 1}^{2}-\frac{\left(\sum_{d=1}^{m_{p}} \bar{B}_{p d 1}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)} . \tag{58}
\end{equation*}
$$

When multiple counts are made each sample day the estimated variance is (Cochran 1977):

$$
\begin{equation*}
\operatorname{Var}\left(\bar{B}_{p}\right)=\left(1-\frac{m_{p}}{D_{p}}\right)\left(\frac{\left.\sum_{d=1}^{m_{p}} \bar{B}_{p d}^{2}-\frac{\left(\sum_{d=1}^{m_{p}} \bar{B}_{p d}\right)^{2}}{m_{p}\left(m_{p}-1\right)}\right)+\frac{1}{D_{p} m_{p}}\left(\sum_{d=1}^{m_{p}} \operatorname{Var}\left(\bar{B}_{p d}\right)\right), ~ \text {, }}{m_{p}}\right) \tag{59}
\end{equation*}
$$

where,

$$
D_{p}=\text { total number of days in period. }
$$

From the interviews (completed-trip), mean total angler hours per boat trip during period $p$ is calculated as:

$$
\begin{equation*}
\bar{e}_{p}=\frac{\sum_{i=1}^{k_{p}} h_{p i}}{k_{p}}, \tag{60}
\end{equation*}
$$

where,

$$
k_{p}=\text { number of angler parties interviewed, }
$$

and estimated variance of mean total angler-hours per boat trip is:

$$
\begin{equation*}
\operatorname{Va} r\left(\bar{e}_{p}\right)=\frac{\sum_{i=1}^{k_{p}} h_{p i}{ }^{2}-\frac{\left(\sum_{i=1}^{k_{p}} h_{p i}\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{61}
\end{equation*}
$$

Estimated boat angler-hours are a product of estimated number of boats and mean total angler-hours per boat trip during period $p$ :

$$
\begin{equation*}
E_{p}=\bar{B}_{p} \bar{e}_{p}, \tag{62}
\end{equation*}
$$

with estimated variance (Goodman 1960):

$$
\begin{equation*}
\operatorname{Var}\left(E_{p}\right)=\bar{B}_{p}^{2} \operatorname{Var} r\left(\bar{e}_{p}\right)+\bar{e}_{p}^{2} \operatorname{Var}\left(\bar{B}_{p}\right)-\operatorname{Var}\left(\bar{e}_{p}\right) \operatorname{Var}\left(\bar{B}_{p}\right) . \tag{63}
\end{equation*}
$$

Estimated boat angler-hours for period $p$, adjusted for non-fishing units ( $\hat{E}_{p}$ ), is calculated using equation (35), and its associated variance is calculated using equation (36).

Instantaneous boat counts.-Estimated number of boat-hours for period $p$ from count $j$ on day $d$ is:

$$
\begin{equation*}
\beta_{p d j}=F_{p} b_{d j} \tag{64}
\end{equation*}
$$

where,

$$
b_{d j}=\text { boats or other units counted during day } d \text { for count } j \text {. }
$$

When $n_{d}>1$, estimated mean number of boat-hours for day $d$ is:

$$
\begin{equation*}
\bar{\beta}_{p d}=\frac{\sum_{j=1}^{n_{d}} \beta_{p d j}}{n_{d}} \tag{65}
\end{equation*}
$$

and the variance is estimated from (20). Estimated mean number of boat-hours for period $p$ is:

$$
\begin{equation*}
\bar{\beta}_{p}=\frac{\sum_{d=1}^{m_{p}} \bar{\beta}_{p d}}{m_{p}} . \tag{66}
\end{equation*}
$$

When one count per sample day is made ( $n_{d}=1$ ), estimated variance of mean number of boathours is calculated from (58). When more than one count per sample day is made, estimated variance of mean number of boat-hours is calculated from (59). In either case, substitute $\beta$ for $B$.

Mean anglers per boat are calculated from the interviews as:

$$
\begin{equation*}
\bar{a}_{p}=\frac{\sum_{i=1}^{k_{p}} a_{p i}}{k_{p}} \tag{67}
\end{equation*}
$$

and the estimated variance of mean anglers per boat is:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{a}_{p}\right)=\frac{\sum_{i=1}^{k_{p}} a_{p i}^{2}-\frac{\left(\sum_{i=1}^{k_{p}} a_{p i}\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{68}
\end{equation*}
$$

Estimated boat angler-hours then is a product of estimated boat-hours and mean anglers per boat:

$$
\begin{equation*}
E_{p}=\bar{\beta}_{p} \bar{a}_{p} \tag{69}
\end{equation*}
$$

and the estimated variance of mean boat angler-hours is:

$$
\begin{equation*}
\operatorname{Var}\left(E_{p}\right)=\bar{\beta}_{p}^{2} \operatorname{Var}\left(\bar{a}_{p}\right)+\bar{a}_{p}^{2} \operatorname{Var} r\left(\bar{\beta}_{p}\right)-\operatorname{Var}\left(\bar{a}_{p}\right) \operatorname{Var}\left(\bar{\beta}_{p}\right) . \tag{70}
\end{equation*}
$$

Estimated boat angler-hours for period $p$, adjusted for non-fishing units ( $\hat{E}_{p}$ ), is calculated using equation (35), and its associated variance is calculated using equation (36).

Instantaneous angler counts.-Since anglers are counted individually, estimated angler-hours for period $p$ from count $j$ on day $d$ are:

$$
\begin{equation*}
\hat{E}_{p d j}=F_{p} A_{d j}, \tag{71}
\end{equation*}
$$

where,

$$
A_{d j}=\text { anglers counted on count } j \text { within day } d .
$$

When $n_{d}>1$, estimated mean angler-hours for day $d$ is:

$$
\begin{equation*}
\bar{E}_{p d}=\frac{\sum_{j=1}^{n_{d}} \hat{E}_{p d j}}{n_{d}}, \tag{72}
\end{equation*}
$$

and the variance is estimated from (29). Estimated mean anger-hours for period $p$ is:

$$
\begin{equation*}
\hat{E}_{p}=\frac{\sum_{d=1}^{m_{p}} \bar{E}_{p d}}{m_{p}} . \tag{73}
\end{equation*}
$$

When one count per sample day is made, the variance of mean angler-hours is estimated from (58). When more than one count per sample day is made, the variance of mean angler-hours is estimated from (59). In either case, substitute anglers for boats.

Estimated angler trips are calculated using equations (37-40) and estimated angler days using equations (41-44). Note that $E_{p}$ and $\operatorname{Var}\left(E_{p}\right)$ are substituted for $\hat{E}_{p}$ and $\operatorname{Var}\left(\hat{E}_{p}\right)$ when only fishing units are counted.

Estimated catch or harvest.-As with daily estimates, estimated catch rate of completed-trip and incompleted-trip interviews (and estimated variance) are treated the same. Also note that the adjustment for non-fishing units is made in the calculation of effort. The estimated catch is calculated as:

$$
\begin{equation*}
\hat{C}_{p}=\hat{E}_{p} R_{p}, \tag{74}
\end{equation*}
$$

where,

$$
R_{p}=\text { appropriate catch rate },
$$

and the estimated variance is:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{C}_{p}\right)=R_{p}^{2} \operatorname{Var}\left(\hat{E}_{p}\right)+\hat{E}_{p}^{2} \operatorname{Var}\left(R_{p}\right)-\operatorname{Var}\left(\hat{E}_{p}\right) \operatorname{Var}\left(R_{p}\right) . \tag{75}
\end{equation*}
$$

Recall that $E_{p}$ and $\operatorname{Var}\left(E_{p}\right)$ are substituted for $\hat{E}_{p}$ and $\operatorname{Var}\left(\hat{E}_{p}\right)$ when only fishing units are counted.

## Targeted Catch, Catch Rates

The design of the Michigan creel program allows for estimation of targeted catch, defined as the catch of a species or group of species from anglers that were targeting that species or group of species. On inland lakes this may not be practical because it is more difficult to discern differences in angling methods. On the Great Lakes, differentiating between targeted catch of salmon and trout and yellow perch is both feasible and necessary, since considerable effort is directed at both groups of species. Estimation of targeted catch and effort becomes particularly important when targeted catch rates are used as an index of species-specific population abundance (Malvestuto 1983).

Targeted effort and targeted catch are calculated similar to estimated catch. Information on targeted effort comes from interviews where anglers are asked what target they are fishing for. Targeted effort rates are calculated using equations (1) through (8) by simply substituting targeted effort for total catch. For example, targeted effort rate [using equation (1)] is estimated by dividing total targeted effort by total effort. For a day, targeted effort rate is multiplied by total estimated effort (from the counts) to obtain a targeted effort estimate for that day.

Similarly, targeted catch rates are also calculated using equations (1) through (8) by substituting targeted catch for total catch. From the interview data then, targeted catch rate is estimated by dividing total targeted catch for a day by total effort for that day. Targeted catch rate is multiplied by total estimated effort (from the counts) to obtain a targeted catch estimate for that day.

Targeted effort and targeted catch are summed across days to estimate targeted effort and targeted catch for a multiple-day period. Targeted catch is then divided by targeted effort to obtain a biologically meaningful estimate of the targeted catch rate for that multiple-day period. As indicated, targeted effort and targeted catch are currently estimated on a daily basis using completed-trip interviews. Conceivably these estimates could be made on a multiple-day basis and with incompleted-trip interviews as well.

## Large Area Estimates With Missing Values

When sampling large areas, such as statistical grids within one of the Great Lakes, catch and effort estimates have been made at sites within the area, such as ports within a grid. These site estimates are then summed to provide a total estimate for the area. However, during some years estimates were not made at all sites. To compensate, these missing values were estimated from years when all sites were sampled. From years when all sites were sampled, a ratio of [all sites within the area to a subset of sites] was calculated. Estimates from the years with missing sites were then expanded from the sampled subset of sites by this ratio to provide a total estimate. This methodology assumes that relationships of the part to the whole are relatively consistent through time. Further evaluation of this method is warranted.

## Discussion

Numerous choices have been presented for estimating angling effort, catch rate and subsequently catch, and each may be estimated by daily or by multiple-day methods. In this section we briefly discuss current and suggested criteria for selecting some of these methods and pertinent biases.

When initiating an angler survey the "best" methods are not always obvious and what is appropriate at one location may not be for another. Making preliminary counts and interviews during a prior representative time period aides in this decision process. For example, prior sampling may show that at a site boats enter the fishery and then disappear from sight, access methods are the obvious choice. Interval counts would be made and completed-trip interviews from angling parties collected. In addition, if numerous angler party interviews are easily collected each sampling day,
daily estimates could be made and concern for effort-catch-rate bias alleviated. Similarly, if angling at a fishing pier is to be estimated and prior sampling shows that anglers are readily available for enumeration and interviewing, either access or roving methods would be appropriate. Instantaneous counts with either completed- or incompleted-trip interviews would be used. Again if numerous interview records may be collected each sampling day, daily estimates are most suitable.

Use of mean-of-ratios catch rate assumes that individual angler's catch rates are relatively constant over time. Since anglers are interviewed prior to completion of their trip, little information is collected regarding the latter portion of their fishing trip. Thus, the assumption of consistent catch rate is important. For example, if anglers in a given population catch more fish towards the end of their fishing trips, mean-of-ratios catch rate will underestimate actual catch rate. Conversely, actual catch rate will be overestimated when fewer fish are caught near the end of fishing trips. When bag limits are effective in limiting catch, Pollock et al. (1997) have shown that mean of ratios catch rate is biased downward. Currently, evaluations of completed- and incompleted-trip interviews from Michigan angler surveys are being conducted to further appraise appropriateness of incompleted-trip interviews from roving surveys.

When collecting an adequate number of interviews each sampling day is difficult, two choices prevail. Either multiple-day estimates may be calculated or sampling effort (number of survey clerks collecting data) increased each day to allow for daily estimates. Again for new angler surveys, prior sampling will assist in determining presence of bias associated with multiple-day estimates and potential of collecting adequate number of interviews for daily estimates.

Choice of targets for targeted effort estimates need to be made pragmatically on a case by case basis. For example, in some situations, categorizing anglers into two categories such as salmon and yellow perch may be entirely reasonable. Since it is highly unlikely, due to salmon fishing gear, that a salmon angler will catch yellow perch, their yellow perch catch rate does not realistically reflect yellow perch catch rate of anglers targeting yellow perch. A similar argument may be made for anglers targeting yellow perch. If anglers targeting chinook salmon readily catch lake trout Salvelinus namaycush or coho salmon Oncorhyncus kisutch however, creating a category for anglers targeting lake trout only (thus excluding anglers targeting chinook salmon), may not satisfactorily reflect lake trout estimates. Yet, estimating targeted catch and targeted catch rates by species may be very appropriate in other situations.

Refinement of angler survey methods is an ongoing process. Equations and methods presented in this paper reflect those most current at the time of publication. Evaluations such as those of catch rate and missing value estimators may require future modification of roving and access site angler survey methods used in Michigan.

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Table 1.-Symbols and subscripts used in equations to estimate angling effort, catch and harvest from Michigan roving and access site angler survey data.

| Variable | Definition |
| :---: | :--- |
| $A$ | anglers counted |
| $B$ | estimated total number of boats from interval counts |
| $\bar{B}$ | estimated mean number of boats from interval counts |
| $\beta$ | estimated total number of boat-hours from instantaneous counts |
| $\bar{\beta}$ | estimated mean number of boat-hours from instantaneous counts |
| $C$ | estimated catch or harvest, usually by species, unadjusted for non-fishing units |
| $\hat{C}$ | estimated catch or harvest, usually by species, adjusted for non-fishing units |
| $D$ | total days in sample period |
| $E$ | estimated angler-hours, unadjusted for non-fishing units |
| $\hat{E}$ | estimated angler-hours, adjusted for non-fishing units |
| $F$ | fishable hours in a stratum |
| $L$ | duration of interval count (hours) |
| $\hat{R}$ | ratio-of-means catch rate estimator |
| $\tilde{\hat{R}}$ | mean of daily ratio of means catch rates |
| $\bar{R}$ | mean-of-ratios catch rate estimator |
| $\tilde{R}$ | mean of daily mean of ratios catch rates |
| $R_{d}$ | represents appropriate catch rate estimator |
| $S$ | estimated angler days |
| $T$ | number of fishing trips taken in a day |
| $U$ | total number of fishing and non-fishing interviews in a stratum |
| $a$ | number of anglers in a party |
| $\bar{a}$ | mean anglers per party |
| $\tilde{\bar{a}}$ | mean of daily anglers per party |
| $b$ | number of boats counted |
| $c$ | catch or harvest in interviews, usually by species |
| $d$ | indexing subscript denoting day within a period |
| $\bar{e}$ | mean boat angler hours per angler trip |
| $\tilde{e}$ | mean of daily boat angler hours per angler trip |
| $\hat{\varepsilon}$ | estimated angler trips |
| $f$ | proportion of fishing boats in interview data set |
| $h$ | total angler hours fished by an interviewed party |
| $i$ | indexing subscript denoting angler or angler party interviewed |
| $j$ | indexing subscript denoting interval or instantaneous counts |
| $k$ | number of anglers interviewed |
| $m$ | number of sampled days within a stratum for which valid estimates can be made |
| $n$ | number of counts |
| $p$ | designates a sample period (typically either all week days or all weekend days within a |
| $t$ | month) |
| $w$ | length of fishing trip (hours) |
| number of sampled days in stratum with fishing interviews |  |
|  |  |

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## Appendix 1.-Equations used to estimate angling effort and catch from Michigan roving and access angler survey data collected prior to 1998.

Angler surveys conducted by MDNR, Fisheries Division prior to 1998 [particularly those referencing Ryckman (1981)] used multiple-day estimate methods, usually days of a day type within a month. Similar to methods given previously in this report, calculated estimates were mode-specific by time period. Multiple time periods and modes were summed for a total estimate. This appendix has been included to more completely document equations used prior to 1998.

When equations in this appendix are the same or similar to those in the main body of this report, those previous equations may be referenced. However, to provide as complete a document as possible, some replication of equations occurs. Symbols and subscripts used in this appendix follow the definitions previously given in Table 1. Additional symbols used this appendix are defined in the appendix text.

## Catch Rate Estimator

The mean of ratios estimator was used for calculating catch per hour for both completed- and incompleted-trip interviews. Angler interviews were collected by angling party. Catch was recorded on interview forms by angling party, and generally no effort was made to separate catch by individual angler. Catch was calculated for period $p$ by angling party as:

$$
\begin{equation*}
\bar{R}_{p}=\frac{\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)}{k_{p}} \tag{76}
\end{equation*}
$$

where,

$$
\begin{aligned}
c_{p i} & =\text { total catch of a particular species by angler party } i, \\
h_{p i} & =\text { total angler-hours fished by angler party } i, \\
k_{p} & =\text { total number of angler parties interviewed },
\end{aligned}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Va} r\left(\bar{R}_{p}\right)=\frac{\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)^{2}-\frac{\left(\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{77}
\end{equation*}
$$

When only one interview was collected for a time period $(k=l)$ and no variance could be calculated for that catch rate, the variance was estimated by:

$$
\begin{equation*}
\operatorname{Var}\left(\overline{\bar{R}}_{p}\right)=\left(\frac{\sqrt{\operatorname{Var}\left(\bar{R}_{2}\right)}}{\bar{R}_{2}} \bar{R}_{p}\right)^{2} \tag{78}
\end{equation*}
$$

where,
$\bar{R}_{2}=$ catch rate for same species during adjacent time period or similar catch rate for another species during adjacent time period.

## Angling Effort

Similar to methods given in previous sections, two types of counts were used: interval and instantaneous. Only one type of count was used during a time period and depending on logistics, either one or multiple counts per sample day were made. Number of counts made per sample day were usually consistent for a given survey.

Interval counts.-Boats were the typical unit counted using this method. However, alternative units may have been counted. Counts were made over a time interval $(L)$, and were usually of consistent length for a survey. Mean number of boats per interval for period $p$ was calculated as:

$$
\begin{equation*}
\bar{\varepsilon}_{p}=\frac{\sum_{j=1}^{m_{p}}\left(\sum_{q=1}^{n} b_{p j q} / n_{p q}\right)}{m_{p}}, \tag{79}
\end{equation*}
$$

where,

$$
\begin{aligned}
b & =\text { number of boats or other units counted over interval } L, \\
n & =\text { number of counts made on a sample day }, \\
m & =\text { number of days sampled during period } p .
\end{aligned}
$$

When one count was made per sample day $(n=1)$, the estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{\varepsilon}_{p}\right)=\frac{\sum_{j=1}^{m_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{m_{p}} b_{p j}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)}, \tag{80}
\end{equation*}
$$

and when multiple counts were made per sample day, the estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{\varepsilon}_{p}\right)=\frac{1}{m_{p}^{2}} \sum_{q=1}^{m_{p}}\left(\frac{\sum_{j=1}^{n_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{n_{p}} b_{p j}\right)^{2}}{n_{p}}}{n_{p}\left(n_{p}-1\right)}\right) . \tag{81}
\end{equation*}
$$

Estimated boats then were calculated as:

$$
\begin{equation*}
\bar{B}_{p}=\bar{\varepsilon}_{p} F_{p}, \tag{82}
\end{equation*}
$$

where,

$$
F_{p}=\text { number of intervals within period } p,
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{B}_{p}\right)=F_{p}^{2} \operatorname{Var}\left(\bar{\varepsilon}_{p}\right) . \tag{83}
\end{equation*}
$$

From the interviews (completed-trip), mean hours per boat trip during period $p$ were calculated as:

$$
\begin{equation*}
\hat{e}_{p}=\frac{\sum_{i=1}^{k_{p}} t_{p i}}{k_{p}} \tag{84}
\end{equation*}
$$

where,

$$
t_{p}=\text { boat trip time },
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{e}_{p}\right)=\frac{\sum_{i=1}^{k_{p}} t_{p i}^{2}-\frac{\left(\sum_{i=1}^{k_{p}} t_{p i}\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{85}
\end{equation*}
$$

Estimated boat-hours then were:

$$
\begin{equation*}
\hat{\bar{B}}_{p}=\bar{B}_{p} \hat{e}_{p} \tag{86}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\overline{\bar{B}}_{p}\right)=\left[\bar{B}_{p}^{2} \operatorname{Var}\left(\hat{e}_{p}\right)\right]+\left[\hat{e}_{p}^{2} \operatorname{Var}\left(\bar{B}_{p}\right)\right] . \tag{87}
\end{equation*}
$$

Using mean anglers per party, $\bar{a}_{p}$ from equation (67), and $\operatorname{Var}\left(\bar{a}_{p}\right)$ from (68), estimated boat angler-hours were calculated as:

$$
\begin{equation*}
E_{p}=\hat{\bar{B}}_{p} \bar{a}_{p} \tag{88}
\end{equation*}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Var}\left(E_{p}\right)=\left[\overline{\bar{B}}_{p}^{2} \operatorname{Var}\left(\bar{a}_{p}\right)\right]+\left[\bar{a}_{p}^{2} \operatorname{Var}\left(\hat{\bar{B}}_{p}\right)\right], \tag{89}
\end{equation*}
$$

When only one interview was collected $(k=1), \operatorname{Var}\left(\hat{e}_{p}\right)$ was estimated as:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{\bar{e}}_{p}\right)=\left(\frac{\sqrt{\operatorname{Var}\left(\hat{e}_{2}\right)}}{\hat{e}_{2}} \hat{e}_{p}\right)^{2}, \tag{90}
\end{equation*}
$$

where,

$$
\begin{aligned}
\hat{e}_{2}= & \text { mean boat hours per boat trip from adjacent time period, and } \operatorname{Var}\left(\hat{\bar{e}}_{p}\right) \\
& \text { substituted for } \operatorname{Var}\left(\hat{e}_{p}\right)
\end{aligned}
$$

In situations where fishing boats were not easily distinguished from non-fishing boats, all boats were counted and all boating parties interviewed with non-fishing boat interviews noted. From the interview data set, proportion of fishing boats was:

$$
\begin{equation*}
f_{p}=\frac{k_{p}}{U_{p}} \tag{91}
\end{equation*}
$$

where,

$$
\begin{aligned}
k_{p} & =\text { total number of angling parties interviewed in time period } p, \\
U_{p} & =\text { total number of angling and non-angling parties interviewed in time period } p,
\end{aligned}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Vâ}\left(f_{p}\right)=\frac{f_{p}\left(1-f_{p}\right)}{U_{p}} . \tag{92}
\end{equation*}
$$

Estimated angling effort for period $p$ was:

$$
\begin{equation*}
\hat{E}_{p}=E_{p} f_{p} \tag{93}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{E}_{p}\right)=\left[E_{p}^{2} \operatorname{Var}\left(f_{p}\right)\right]+\left[f_{p}^{2} \operatorname{Var}\left(E_{p}\right)\right] . \tag{94}
\end{equation*}
$$

Instantaneous boat counts.-Boats are the units discussed in this section, however other units such as trailers or ice shanties were common. Methods described in this section were used for any unit which could represent more than one angler. Mean number of boats during period $p$ were:

$$
\begin{equation*}
\bar{b}_{p}=\frac{\sum_{j=1}^{m_{p}}\left(\sum_{q=1}^{n}\left(b_{p j q}\right) / n_{p q}\right.}{m_{p}} \tag{95}
\end{equation*}
$$

where,

$$
b=\text { number of boats or other units counted at a given instant. }
$$

When one count was made per day ( $n=1$ ), estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{b}_{p}\right)=\frac{\sum_{j=1}^{m_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{m_{p}} b_{p j}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)}, \tag{96}
\end{equation*}
$$

and when multiple counts per day were made, estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{b}_{p}\right)=\frac{1}{m_{p}^{2}} \sum_{q=1}^{m_{p}}\left(\frac{\sum_{j=1}^{n_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{n_{p}} b_{p j}\right)^{2}}{n_{p}}}{n_{p}\left(n_{p}-1\right)}\right) \tag{97}
\end{equation*}
$$

Estimated boat-hours were calculated as:

$$
\begin{equation*}
\bar{\beta}_{p}=\bar{b}_{p} F_{p} \tag{98}
\end{equation*}
$$

where,

$$
F_{p}=\text { number of hours in period } p,
$$

and estimated variance as:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{\beta}_{p}\right)=F_{p}^{2} \operatorname{Var}\left(\bar{b}_{p}\right) . \tag{99}
\end{equation*}
$$

Recall that mean length of a party fishing trip for period $p\left(\hat{e}_{p}\right)$ is calculated using equations (84) and (85). Estimated boat angling effort from instantaneous counts then was:

$$
\begin{equation*}
E_{p}=\bar{\beta}_{p} \hat{e}_{p} \tag{100}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(E_{p}\right)=\left[\bar{\beta}_{p}^{2} \operatorname{Var}\left(\hat{e}_{p}\right)\right]+\left[\hat{e}_{p}^{2} \operatorname{Var}\left(\bar{\beta}_{p}\right)\right] . \tag{101}
\end{equation*}
$$

In the event only one interview was collected $(k=1), \operatorname{Var}\left(\overline{\bar{e}}_{p}\right)$ was substituted for $\operatorname{Var}\left(\hat{e}_{p}\right)$.
When fishing boats were not easily distinguished from non-fishing boats, all boating parties were interviewed with non-fishing parties noted and total boat effort (fishing and non-fishing) adjusted using equations (91-94). Similarly, when ice shanties were counted, and occupied and unoccupied shanties were not distinguishable, equations (91) and (92) were used to estimate a correction factor for occupied shanties. Typically, randomly selected days and times, within the period $p$, were selected and proportion of occupied shanties in total shanty count was estimated. This proportion and its variance were substituted in equations (93) and (94) to estimate shanty angling effort.

Instantaneous angler counts.-Anglers were counted directly and counts did not require expansion other than by sample hours within the period. Mean number of anglers per count was:

$$
\begin{equation*}
\alpha_{p}=\frac{\sum_{j=1}^{m_{p}}\left(\sum_{q=1}^{n} A_{p j q} / n_{p q}\right.}{m_{p}} . \tag{102}
\end{equation*}
$$

When one count was made per sample day $(n=l)$, estimated variance was:

$$
\begin{equation*}
\operatorname{Vâr}\left(\alpha_{p}\right)=\frac{\sum_{j=1}^{m_{p}} A_{p j}^{2}-\frac{\left(\sum_{j=1}^{m_{p}} A_{p j}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)}, \tag{103}
\end{equation*}
$$

and when multiple-counts per day were made, estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\alpha_{p}\right)=\frac{1}{m_{p}^{2}} \sum_{q=1}^{m_{p}}\left(\frac{\sum_{j=1}^{n_{p}} A_{p j}^{2}-\frac{\left(\sum_{j=1}^{n_{p}} A_{p j}\right)^{2}}{n_{p}}}{n_{p}\left(n_{p}-1\right)}\right) \text {. } \tag{104}
\end{equation*}
$$

Estimated angler hours then were:

$$
\begin{equation*}
\hat{E}_{p}=\alpha_{p} F_{p}, \tag{105}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{E}_{p}\right)=F_{p}^{2} \operatorname{Var}\left(\alpha_{p}\right) . \tag{106}
\end{equation*}
$$

Estimated angling trips.-Mean length of angler trip $\left(\bar{t}_{p}\right)$ from the interview data set comes from equation (37) with variance equation $\left(\operatorname{Var}\left(\bar{t}_{p}\right)\right)$ from equation (38). Estimated angling trips for period $p$ then were:

$$
\begin{equation*}
\hat{\varepsilon}_{p}=\frac{\hat{E}_{p}}{\bar{t}_{p}} \tag{107}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{\varepsilon}_{p}\right)=\hat{\varepsilon}_{p}^{2}\left[\frac{\operatorname{Var}\left(\hat{E}_{p}\right)}{\hat{E}_{p}^{2}}+\frac{\operatorname{Var}\left(\bar{t}_{p}\right)}{\bar{t}_{p}^{2}}\right] . \tag{108}
\end{equation*}
$$

When only one interview was collected $(k=1), \operatorname{Var}\left(\bar{t}_{p}\right)$ was estimated as:

$$
\begin{equation*}
\operatorname{Var}\left(\tilde{\bar{t}}_{p}\right)=\left(\frac{\sqrt{\operatorname{Var}\left(\bar{t}_{2}\right)}}{\bar{t}_{2}} \bar{t}_{p}\right)^{2}, \tag{109}
\end{equation*}
$$

where,

$$
\bar{t}_{2}=\text { mean length of angler trip from adjacent time period, }
$$

and $\operatorname{Var}\left(\tilde{\bar{t}}_{p}\right)$ was substituted for $\operatorname{Var}\left(\bar{t}_{p}\right)$. Similarly, when only angling units were counted $E_{p}$ and $\operatorname{Var}\left(E_{p}\right)$ were substituted for $\hat{E}_{p}$ and $\operatorname{Var}\left(\hat{E}_{p}\right)$.

## Estimated Catch or Harvest

Estimated catch was the product of estimated angling effort (note that effort has been adjusted to remove non-angling effort) and mean-of-ratios catch rate:

$$
\begin{equation*}
\hat{C}_{p}=\hat{E}_{p} \bar{R}_{p}, \tag{110}
\end{equation*}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Va} r\left(\hat{C}_{p}\right)=\left[\hat{E}_{p}^{2} \operatorname{Var} r\left(\bar{R}_{p}\right)\right]+\left[\bar{R}_{p}^{2} \operatorname{Var}\left(\hat{E}_{p}\right)\right] . \tag{111}
\end{equation*}
$$

In the event only angling units were counted, $E_{p}$ and $\operatorname{Var}\left(E_{p}\right)$ was substituted for $\hat{E}_{p}$ and $\operatorname{Var}\left(\hat{E}_{p}\right)$, and when only one interview was collected $(k=1), \operatorname{Var}\left(\overline{\bar{R}}_{p}\right)$ was substituted for $\operatorname{Var}\left(\bar{R}_{p}\right)$.

See attached Appendix 1 errata, pages 29-35.

Errata - This revision includes addition of equation (87.1), and corrections to equations (90), (100), and (101), and associated text.

## Appendix 1.-Equations used to estimate angling effort and catch from Michigan roving and access angler survey data collected prior to 1998.

Angler surveys conducted by MDNR, Fisheries Division prior to 1998 [particularly those referencing Ryckman (1981)] used multiple-day estimate methods, usually days of a day type within a month. Similar to methods given previously in this report, calculated estimates were mode-specific by time period. Multiple time periods and modes were summed for a total estimate. This appendix has been included to more completely document equations used prior to 1998.

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## Catch Rate Estimator

The mean of ratios estimator was used for calculating catch per hour for both completed- and incompleted-trip interviews. Angler interviews were collected by angling party. Catch was recorded on interview forms by angling party, and generally no effort was made to separate catch by individual angler. Catch was calculated for period $p$ by angling party as:

$$
\begin{equation*}
\bar{R}_{p}=\frac{\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)}{k_{p}}, \tag{76}
\end{equation*}
$$

where,

$$
\begin{aligned}
c_{p i} & =\text { total catch of a particular species by angler party } i, \\
h_{p i} & =\text { total angler-hours fished by angler party } \mathrm{i}, \\
k_{p} & =\text { total number of angler parties interviewed, }
\end{aligned}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Vâ} r\left(\bar{R}_{p}\right)=\frac{\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)^{2}-\frac{\left(\sum_{i=1}^{k_{p}}\left(c_{p i} / h_{p i}\right)\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} \tag{77}
\end{equation*}
$$

When only one interview was collected for a time period $(k=l)$ and no variance could be calculated for that catch rate, the variance was estimated by:

$$
\begin{equation*}
\operatorname{Var}\left(\overline{\bar{R}}_{p}\right)=\left(\frac{\sqrt{\operatorname{Var}\left(\bar{R}_{2}\right)}}{\bar{R}_{2}} \bar{R}_{p}\right)^{2} \tag{78}
\end{equation*}
$$

where,

$$
\begin{aligned}
\bar{R}_{2}= & \text { catch rate for same species during adjacent time period or similar catch rate for } \\
& \text { another species during adjacent time period. }
\end{aligned}
$$

## Angling Effort

Similar to methods given in previous sections, two types of counts were used: interval and instantaneous. Only one type of count was used during a time period and depending on logistics, either one or multiple counts per sample day were made. Number of counts made per sample day were usually consistent for a given survey.

Interval counts.-Boats were the typical unit counted using this method. However, alternative units may have been counted. Counts were made over a time interval ( $L$ ), and were usually of consistent length for a survey. Mean number of boats per interval for period $p$ was calculated as:

$$
\begin{equation*}
\bar{\varepsilon}_{p}=\frac{\sum_{j=l}^{m_{p}}\left(\sum_{q=l}^{n} b_{p j q} / n_{p q}\right)}{m_{p}}, \tag{79}
\end{equation*}
$$

where,

$$
\begin{aligned}
\mathrm{b} & =\text { number of boats or other units counted over interval } \mathrm{L}, \\
\mathrm{n} & =\text { number of counts made on a sample day } \\
\mathrm{m} & =\text { number of days sampled during period } \mathrm{p} .
\end{aligned}
$$

When one count was made per sample day $(n=1)$, the estimated variance was:

$$
\begin{equation*}
\operatorname{Varr}\left(\bar{\varepsilon}_{p}\right)=\frac{\sum_{j=1}^{m_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{m_{p}} b_{p j}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)}, \tag{80}
\end{equation*}
$$

and when multiple counts were made per sample day, the estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{\varepsilon}_{p}\right)=\frac{1}{m_{p}^{2}} \sum_{q=1}^{m_{p}}\left(\frac{\sum_{j=1}^{n_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{n_{p}} b_{p j}\right)^{2}}{n_{p}}}{n_{p}\left(n_{p}-1\right)}\right) \tag{81}
\end{equation*}
$$

Estimated boats then were calculated as:

$$
\begin{equation*}
\bar{B}_{p}=\bar{\varepsilon}_{p} F_{p} \tag{82}
\end{equation*}
$$

where,

$$
F_{p}=\text { number of intervals within period } p,
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{B}_{p}\right)=F_{p}^{2} \operatorname{Var}\left(\bar{\varepsilon}_{p}\right) . \tag{83}
\end{equation*}
$$

From the interviews (completed-trip), mean hours per boat trip during period $p$ were calculated as:

$$
\begin{equation*}
\hat{e}_{p}=\frac{\sum_{i=l}^{k_{p}} t_{p i}}{k_{p}} \tag{84}
\end{equation*}
$$

where,

$$
t_{p}=\text { boat trip time, }
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{e}_{p}\right)=\frac{\sum_{i=1}^{k_{p}} t_{p i}^{2}-\frac{\left(\sum_{i=1}^{k_{p}} t_{p i}\right)^{2}}{k_{p}}}{k_{p}\left(k_{p}-1\right)} . \tag{85}
\end{equation*}
$$

Estimated boat-hours then were:

$$
\begin{equation*}
\hat{\bar{B}}_{p}=\bar{B}_{p} \hat{e}_{p} \tag{86}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\overline{\bar{B}}_{p}\right)=\left[\bar{B}_{p}^{2} \operatorname{Var}\left(\hat{e}_{p}\right)\right]+\left[\hat{e}_{p}^{2} \operatorname{Var}\left(\bar{B}_{p}\right)\right] . \tag{87}
\end{equation*}
$$

When only one interview was collected $(k=1), \operatorname{Var}\left(\hat{e}_{p}\right)$ was estimated as:

$$
\begin{equation*}
\operatorname{Var}\left(\overline{\bar{e}}_{p}\right)=\left(\frac{\sqrt{\operatorname{Var}\left(\hat{e}_{2}\right)}}{\hat{e}_{2}} \hat{e}_{p}\right)^{2}, \tag{87.1}
\end{equation*}
$$

where,

$$
\hat{e}_{2}=\text { mean boat hours per boat trip from adjacent time period, }
$$

and $\operatorname{Var}\left(\overline{\bar{e}}_{p}\right)$ substituted for $\operatorname{Var}\left(\hat{e}_{p}\right)$.
Using mean anglers per party, $\bar{a}_{p}$ from equation (67), and $\operatorname{Var}\left(\bar{a}_{p}\right)$ from (68), estimated boat angler-hours were calculated as:

$$
\begin{equation*}
E_{p}=\hat{\bar{B}}_{p} \bar{a}_{p} \tag{88}
\end{equation*}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Var}\left(E_{p}\right)=\left[\overline{\bar{B}}_{p}^{2} \operatorname{Var}\left(\bar{a}_{p}\right)\right]+\left[\bar{a}_{p}^{2} \operatorname{Var}\left(\overline{\bar{B}}_{p}\right)\right], \tag{89}
\end{equation*}
$$

When only one interview was collected $(k=1), \operatorname{Var}\left(\bar{a}_{p}\right)$ was estimated as:

$$
\begin{equation*}
\operatorname{Vâr}\left(\overline{\bar{a}}_{p}\right)=\left(\frac{\sqrt{\operatorname{Var} r\left(\bar{a}_{2}\right)}}{\bar{a}_{2}} \bar{a}_{p}\right)^{2}, \tag{90}
\end{equation*}
$$

where,

$$
\bar{a}_{2}=\text { mean boat hours per boat trip from adjacent time period, }
$$

and $\operatorname{Var} r\left(\overline{\bar{a}}_{p}\right)$ substituted for $\operatorname{Var}\left(\bar{a}_{p}\right)$.
In situations where fishing boats were not easily distinguished from non-fishing boats, all boats were counted and all boating parties interviewed with non-fishing boat interviews noted. From the interview data set, proportion of fishing boats was:

$$
\begin{equation*}
f_{p}=\frac{k_{p}}{U_{p}}, \tag{91}
\end{equation*}
$$

where,

$$
\begin{aligned}
k_{p} & =\text { total number of angling parties interviewed in time period } p, \\
U_{p} & =\text { total number of angling and non-angling parties interviewed in time period } p,
\end{aligned}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Vâ}\left(f_{p}\right)=\frac{f_{p}\left(1-f_{p}\right)}{U_{p}} . \tag{92}
\end{equation*}
$$

Estimated angling effort for period $p$ was:

$$
\begin{equation*}
\hat{E}_{p}=E_{p} f_{p}, \tag{93}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{E}_{p}\right)=\left[E_{p}^{2} \operatorname{Var}\left(f_{p}\right)\right]+\left[f_{p}^{2} \operatorname{Var}\left(E_{p}\right)\right] . \tag{94}
\end{equation*}
$$

Instantaneous boat counts.-Boats are the units discussed in this section, however other units such as trailers or ice shanties were common. Methods described in this section were used for any unit which could represent more than one angler. Mean number of boats during period $p$ were:

$$
\begin{equation*}
\bar{b}_{p}=\frac{\sum_{j=l}^{m_{p}}\left(\sum_{q=1}^{n}\left(b_{p j q}\right) / n_{p q}\right.}{m_{p}} \tag{95}
\end{equation*}
$$

where,

$$
b=\text { number of boats or other units counted at a given instant. }
$$

When one count was made per day ( $n=1$ ), estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{b}_{p}\right)=\frac{\sum_{j=1}^{m_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{m_{p}} b_{p j}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)}, \tag{96}
\end{equation*}
$$

and when multiple counts per day were made, estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{b}_{p}\right)=\frac{1}{m_{p}^{2}} \sum_{q=1}^{m_{p}}\left(\frac{\sum_{j=1}^{n_{p}} b_{p j}^{2}-\frac{\left(\sum_{j=1}^{n_{p}} b_{p j}\right)^{2}}{n_{p}}}{n_{p}\left(n_{p}-1\right)}\right) \tag{97}
\end{equation*}
$$

Estimated boat-hours were calculated as:

$$
\begin{equation*}
\bar{\beta}_{p}=\bar{b}_{p} F_{p}, \tag{98}
\end{equation*}
$$

where,

$$
F_{p}=\text { number of hours in period } p,
$$

and estimated variance as:

$$
\begin{equation*}
\operatorname{Var}\left(\bar{\beta}_{p}\right)=F_{p}^{2} \operatorname{Var}\left(\bar{b}_{p}\right) . \tag{99}
\end{equation*}
$$

Recall that mean number of anglers per fishing party for period $p\left(\bar{a}_{p}\right)$ is calculated using equations (67) and (68). Estimated boat angling effort from instantaneous counts then was:

$$
\begin{equation*}
E_{p}=\bar{\beta}_{p} \bar{a}_{p} \tag{100}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Va} r\left(E_{p}\right)=\left[\bar{\beta}_{p}^{2} \operatorname{Var}\left(\bar{a}_{p}\right)\right]+\left[\bar{a}_{p}^{2} \operatorname{Va} r\left(\bar{\beta}_{p}\right)\right] . \tag{101}
\end{equation*}
$$

In the event only one interview was collected $(k=1), \operatorname{Var}\left(\overline{\bar{a}}_{p}\right)$ was substituted for $\operatorname{Var}\left(\bar{a}_{p}\right)$.
When fishing boats were not easily distinguished from non-fishing boats, all boating parties were interviewed with non-fishing parties noted and total boat effort (fishing and non-fishing) adjusted using equations (91-94). Similarly, when ice shanties were counted, and occupied and unoccupied shanties were not distinguishable, equations (91) and (92) were used to estimate a correction factor for occupied shanties. Typically, randomly selected days and times, within the period $p$, were selected and proportion of occupied shanties in total shanty count was estimated. This proportion and its variance were substituted in equations (93) and (94) to estimate shanty angling effort.

Instantaneous angler counts.-Anglers were counted directly and counts did not require expansion other than by sample hours within the period. Mean number of anglers per count was:

$$
\begin{equation*}
\alpha_{p}=\frac{\sum_{j=1}^{m_{p}}\left(\sum_{q=1}^{n} A_{p j q} / n_{p q}\right)}{m_{p}} \tag{102}
\end{equation*}
$$

When one count was made per sample day $(n=1)$, estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\alpha_{p}\right)=\frac{\sum_{j=1}^{m_{p}} A_{p j}^{2}-\frac{\left(\sum_{j=1}^{m_{p}} A_{p j}\right)^{2}}{m_{p}}}{m_{p}\left(m_{p}-1\right)} \tag{103}
\end{equation*}
$$

and when multiple-counts per day were made, estimated variance was:

$$
\begin{equation*}
\operatorname{Var}\left(\alpha_{p}\right)=\frac{1}{m_{p}^{2}} \sum_{q=1}^{m_{p}}\left(\frac{\sum_{j=1}^{n_{p}} A_{p j}^{2}-\frac{\left(\sum_{j=1}^{n_{p}} A_{p j}\right)^{2}}{n_{p}}}{n_{p}\left(n_{p}-1\right)}\right) \tag{104}
\end{equation*}
$$

Estimated angler hours then were:

$$
\begin{equation*}
\hat{E}_{p}=\alpha_{p} F_{p} \tag{105}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{E}_{p}\right)=F_{p}^{2} \operatorname{Var}\left(\alpha_{p}\right) \tag{106}
\end{equation*}
$$

Estimated angling trips.-Mean length of angler trip $\left(\bar{t}_{p}\right)$ from the interview data set comes from equation (37) with variance equation ( $\operatorname{Var}\left(\bar{t}_{p}\right)$ ) from equation (38). Estimated angling trips for period $p$ then were:

$$
\begin{equation*}
\hat{\varepsilon}_{p}=\frac{\hat{E}_{p}}{\bar{t}_{p}} \tag{107}
\end{equation*}
$$

with estimated variance:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{\varepsilon}_{p}\right)=\hat{\varepsilon}_{p}^{2}\left[\frac{\operatorname{Var}\left(\hat{E}_{p}\right)}{\hat{E}_{p}^{2}}+\frac{\operatorname{Var}\left(\bar{t}_{p}\right)}{\bar{t}_{p}^{2}}\right] . \tag{108}
\end{equation*}
$$

When only one interview was collected $(k=1), \operatorname{Var}\left(\bar{t}_{p}\right)$ was estimated as:

$$
\begin{equation*}
\operatorname{Var}\left(\tilde{\bar{t}}_{p}\right)=\left(\frac{\sqrt{\operatorname{Var}\left(\bar{t}_{2}\right)}}{\bar{t}_{2}} \bar{t}_{p}\right)^{2} \tag{109}
\end{equation*}
$$

where,

$$
\bar{t}_{2}=\text { mean length of angler trip from adjacent time period, }
$$

and $\operatorname{Var}\left(\overline{\bar{t}}_{p}\right)$ was substituted for $\operatorname{Var}\left(\bar{t}_{p}\right)$. Similarly, when only angling units were counted $E_{p}$ and $\operatorname{Var}\left(E_{p}\right)$ were substituted for $\hat{E}_{p}$ and $\operatorname{Var}\left(\hat{E}_{p}\right)$.

## Estimated Catch or Harvest

Estimated catch was the product of estimated angling effort (note that effort has been adjusted to remove non-angling effort) and mean-of-ratios catch rate:

$$
\begin{equation*}
\hat{C}_{p}=\hat{E}_{p} \bar{R}_{p}, \tag{110}
\end{equation*}
$$

with estimated variance as:

$$
\begin{equation*}
\operatorname{Var}\left(\hat{C}_{p}\right)=\left[\hat{E}_{p}^{2} \operatorname{Var}\left(\bar{R}_{p}\right)\right]+\left[\bar{R}_{p}^{2} \operatorname{Var}\left(\hat{E}_{p}\right)\right] . \tag{111}
\end{equation*}
$$

In the event only angling units were counted, $E_{p}$ and $\operatorname{Va} r\left(E_{p}\right)$ was substituted for $\hat{E}_{p}$ and $\operatorname{Var}\left(\hat{E}_{p}\right)$, and when only one interview was collected $(k=1), \operatorname{Var}\left(\overline{\bar{R}}_{p}\right)$ was substituted for $\operatorname{Var}\left(\bar{R}_{p}\right)$.

